Wage Inequality and Education Choices in Latin America: a General Equilibrium Approach*

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Abstract

From being linear at the end of the 1980s, during the 1990s the wage profile in Latin America became a convex function of the level of education: the wage gap between Higher and Intermediate Education increased and the one between Intermediate and Basic Education declined. The double change in the wage differentials was driven by a significant drop in the mean wage at Intermediate that fell below the level at the end of the 1980s. The convexification has important implications for human capital investments and the evolution of wage inequality. Yet, no empirical study has developed a framework within which to study its determinants. This paper provides such a framework. I develop and simulate a dynamic general equilibrium model of savings and educational choices under credit constraints and uninsurable risk in which ability is an important component of individuals' earnings. I estimate the parameters of the model using micro data from Mexico. I specify a production function that allows for complementarities between Intermediate and Higher Education. I find that Intermediate Education is more complementary with Higher than with Basic Education. The results show that the convexification is not the result of a self-selection process of high ability individuals into Higher Education, but rather came about through changes in the prices of schooling due to changes in its supply. The wage profile convexified because of an increase in the relative supply of Intermediate Education in response to a skill biased technological change. The simulations identify two main determinants of the convexification. First, the complementarities in the production function are responsible for the double change in the wage differentials: since Higher Education is more complementary with Intermediate than is Basic Education, an increase in the supply at Intermediate increases the relative return to Higher with respect to Basic Education and further decreases the relative return to Intermediate versus Basic Education. Second, the level of the credit constraints determines the extent of the supply increase at Intermediate Education and therefore the change in the wage at this level. There is a borrowing threshold below which the increase in the supply at Intermediate is not big enough to produce the sizable drop in the wage that is observed in the data. The model is able to reproduce both the double change in the wage differentials and the decline in the wage at Intermediate that characterize the change in wage inequality in Mexico between 1987 and 2002.

Key Words: Latin America, Wage Inequality, Investment in Education, General Equilibrium.
JEL Codes: J23, J24, J31, C68.

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1 Introduction

In the decade of the 1990s Latin American Countries (LACs) have been characterized by a process of educational expansion towards Intermediate Education and significant changes in the wage differentials between education levels. Compulsory education became almost universal and the share of individuals that completed Secondary and High School increased from around 32 to 48 per cent.[1] Graduation rates increased also at Higher Education: the proportion of College graduates went from an average of 13 per cent in 1987 to an average of 18 per cent in the year 2002.[2] At the same time, the wage differential between Higher and Intermediate Education increased and the one between Intermediate and Basic Education declined. The log wage difference between Intermediate and Higher Education increased by as much as 72 percent in Mexico and the log wage difference between Intermediate and Basic Education declined by as much as 40 percent in Brazil.

The result of the double change in the relative wages at Higher and at Intermediate Education is shown in figures 1 to 3. From a linear relationship between mean log wages and the level of education at the end of the 1980s, the wage profile became convex with proportional increases in education translating into higher than proportional increases in income for the more educated. The changes in relative wages were driven by a decline in the mean wage at Intermediate that fell below the value at the end of the 1980s. Between 1987 and 2002 the mean log wage at Intermediate decreased in real terms by three and two per cent in Brazil and Colombia and by as much as five per cent in Mexico.

The convexification of the wage profile has received almost no attention.[3] The empirical literature on LACs has focused primarily on the increase in the premium to Higher Education.[4] Most of the studies have found evidence of an increased demand for skilled labour in production and concluded that a skill-biased technological change (SBTC - hereafter) was the driving force of the increase in the premium to College. Yet, the characterizing feature of the changes in wage inequality and possibly one of the main reasons for the increase in the College premium was the

[1] In the year 1987 the proportion of the adult population with completed Secondary education was 32 per cent in Brazil, 34 per cent in Colombia and 30 per cent in Mexico. In the year 2002 these proportions increased, respectively, to 55, 43, 45 per cent in each of the three countries.

[2] In the year 1987 the proportion of College graduates among the adult population was around 12 per cent in Brazil, 17 per cent in Colombia and 10 per cent in Mexico. In the year 2002 these proportions increased to, respectively, 14, 21 and 17 per cent.

[3] Interestingly, a simple look at the returns to education estimated for East Asian countries (see, for Vietnam, Liu (2006)) shows that the wage profile did also convexified there during the 1990s. However, as for Latin America, most empirical studies focused on the increase in the returns to Higher Education.

[4] One notable exception is Bouillon, Legovini and Lustig (2005) that discuss the unequalizing effect of the convexification in Mexico.

decline in the *level* of the wage at Intermediate. A SBTC on its own could explain the changes in the wage differentials but not in the level of wages.

The mean wages in figures 1 to 3 are a combination of three main components: the return to observables and unobservables characteristics (unobserved ability or quality of the workers) of those completing each level of education and the price of education, which is the actual return to completion of a given education level independently of any observable and unobservable individual characteristic. Therefore, the change of the wage profile from linear to convex could be produced for three main reasons: a change in the composition on observables and their returns, a change in the composition of unobserved ability of those completing different levels of education or a change in the prices of education.[5]

In the first case, the convexification could be the result of changes in the composition by and in the returns to observable characteristics that are relevant for wage determination, such as cohort, gender, sector of employment and work experience. Unreported graphs show that this is not the explanation: the double change in the wage differential driven by the drop in the wage at Intermediate Education remains after controlling for observables and their interaction with the level of education.

In the second case, the change in the shape of the wage function could be produced by a self-selection process of high ability into Higher Education. If ability is an important component of earnings and high-ability individuals self-select into Higher Education, an educational expansion at Intermediate could naturally lead to a decline in the mean wage at this level and an increase in the mean wage at Higher Education. To the best of our knowledge, Binelli, Meghir and Menezes-Filho (2007) (BMM - hereafter) is the only study that investigates the role of changes in the ability composition on the returns to different level of education in a Latin American country. They estimate the changes in the ability distribution by level of education and birth cohort during the educational expansion in Brazil in the 1990s. They find that the changes in composition produced a parallel shift of the wage profile rather than a change in its shape.

In the third case, the production technology and the interactions between the demand and the supply of skill are of crucial importance: any change in the demand of skilled and unskilled labour would affect the incentives to invest in human capital and return a value for the skill prices that reflect the degree of complementarity and substitutability between the different production

[5] These three explanations take as given a set of labor market institutions, such as the level of the minimum wage and the degree of unionization. I abstract from changes in labor market institutions since the available evidence for Latin America shows that these changes on their own could not explain the convexification. I will discuss the empirical evidence in details for the case of Mexico.

inputs. There are no empirical studies of wage inequality in LACs that allow for endogenous education prices, so nothing is known on the importance of the general equilibrium price effects of changes in the demand and the supply of skill.

The aim of this paper is to develop a framework to study the determinants of the convexification. Given the alternative explanations described above, the framework will have two key characteristics: it will be of general equilibrium with the inclusion of the demand and the supply of skills and it will be able to account for self-selection on ability into education.

I focus on Mexico that offers a good case study for two main reasons. First, there exist panel data on earnings that allow to control for unobserved ability for a first assessment of the role of changes in the composition on unobservables to produce the convex wage profile. Figure 4 presents the hourly wages by education estimated from a fixed effect regression that considers all individuals aged between 15 and 60. Once ability is controlled for the convexification is even more evident.

Second, there is established evidence of a SBTC due to a series of labour market and trade reforms that resulted into an increased demand for skilled labour in the mid 1990s. A SBTC could have been an important determinant of the convexification by inducing an increase in the demand for skilled labor in production. Then, the supply reaction will depend on the factors that affect educational choices. Among those, two important ones are the level of credit constraints and the degree of earnings' uncertainty that individuals face. A change in the level of credit constraints and risk could be crucial to determine the size of the supply response and therefore the extent of the convexification.

I develop an incomplete market, dynamic, general equilibrium model of savings and educational choices where the interest rate is taken as given and the education prices are the equilibrium outcomes of changes in the supply and in the demand for skills. My model has two key features. First, as in Cunha (2006), educational (and savings) choices are taken by altruistic parents that face constraints on resources. The economy is made of overlapping generations of parents and children with three distinct market failures: parents cannot buy insurance against their own earnings risk, cannot borrow against future earnings of their children and face credit constraints. In each period parental resources are given by a stochastic labor income and the stocks of a risk-free asset. Credit constraints are modelled as a limit to the amount that individuals can borrow ranging from zero to the maximum amount that an individual can repay with certainty over the lifecycle, which is a function of the level of education, an insurable i.i.d.

shock and an endowment of ability received at birth and perfectly transmitted over generations. By having ability as an important determinant of wages, the model endogenously generates a process of self-selection of higher ability into Higher Education.

Second, the production function allows for complementarities between Intermediate and Higher Education. I assume that three education levels, Basic, Intermediate and Higher Education, build up the economy human capital endowment and I allow the elasticity of substitution (ES) between Intermediate and Basic Education to differ from the one between Intermediate and Higher Education. Intermediate and Higher Education define the skilled human capital. I estimate the production function using micro data from Mexico between 1987 and 2002. I find that the substitution elasticities are consistent with the complementarity between Intermediate and Higher Education. A key implication is that the growth of the relative supply of Intermediate Education increases the relative demand for Higher Education and therefore its marginal product while it decreases the relative return of Intermediate with respect to Basic Education. Together with the parameters governing the ES, I estimate the value of the factor shares of skilled and unskilled labour in 1987. Following Heckman, Lochner and Taber (1998) (HLT - hereafter), I model a SBTC as an increase in the share of skilled labor in production.

I take as a benchmark an economy with the credit constraints at the maximum level of no possible borrowing and the share of skilled labour fixed at the value estimated in 1987 before any SBTC took place. I estimate the production function and the initial distributions of ability, wealth and education with micro-data from Mexico. I solve and compute the transitional dynamics and the steady state of the model. To highlight the most important results, I focus on the steady state.

The main findings of the simulations are three. First, and most importantly, the convexification is a result of the increase in the relative supply at Intermediate level in response to a SBTC. Both the general equilibrium feature of the model and the SBTC are necessary to this result. If the supply by education had been constant at the level observed at the end of the 1980s, a SBTC would have produced an increase in both the relative return to Higher and to Intermediate Education. At the same time, in the absence of any SBTC, the economy in general equilibrium would have converged to a steady state wage profile that is a linear function of the level of education. The complementarity between Intermediate and Higher Education drives this result. In addition to the standard supply effect, an increase in the supply of Intermediate that is bigger in magnitude than the one at Higher Education further decreases the relative return to

Intermediate with respect to Basic Education and increases the relative return to Higher with respect to Basic Education.

Second, the level of the credit constraints determines the extent of the supply increase at Intermediate and therefore the size of the decline in the price at this level. There is a borrowing threshold below which the size of the supply increase at Intermediate is not enough to produce a drop in the equilibrium price at this level so that it falls below the value at baseline. Third, an increase in the level of uninsurable risk produces an equilibrium prices' schedule that is "not enough" convex to match what is observed in the data: a high level of risk would have prevented the sizable supply increase at Intermediate and the drop of the price at this level that characterize the convexification.

Overall, the results suggest that the convexification is the result of changes in the prices of education driven by an increase in the supply of Intermediate Education in reaction to a SBTC. The changes in the supply of education result into significant shifts in the ability distribution by education. However, they are an outcome rather than the driving force of the convexification.

The results of this paper provide an empirical test of the steady-state theory of equilibrium wage functions developed by Mookherjee and Ray (2007). In a model of savings and occupational choices without any stochastic shock and individual heterogeneity, Mookherjee and Ray derive the theoretical implication of a convex relationship between the skill-intensity of an occupation and its marginal rate of return.

The empirical results in this paper show that the shape of the equilibrium wage function depends crucially on the interaction between the features of the production function and on the factors that have a direct impact on educational choices, in particular, on the the degree of substitutability between aggregate labor inputs and on level of credit constraints that individuals face.

1.1 Literature review

Several contributions are related to this paper. They can be classified in two main categories: the empirical literature on education and wages in Latin America and the literature that uses dynamic general equilibrium (GE) models to study the changes in wage inequality and their determinants.

With respect to the literature on wages and education in LACs, both the object of interest and the approach taken in this paper are rather novel. Up to now all the contributions have been focusing on the increase in the premium to Higher Education rather than on the convexification of the wage profile and have adopted a partial equilibrium approach explaining the rise in the premium with changes in the "demand" or in the "supply" of education.

The "supply-side" literature focuses on financial constraints on educational choices and self-selection on ability into Higher Education as two alternative determinants of the increase in the relative wage of Higher versus Intermediate Education. If ability is an important component of wages, the premium to Higher Education might simply reflect a composition effect of higher ability into Higher Education. Alternatively, a decrease in the tightness of the financial constraints at the Intermediate level with the constraints remaining binding at Higher Education could also have produced the increase in the skill premium by allowing the supply to increase more at Intermediate than at Higher Education.

The main contribution for Mexico is Jacoby and Skoufias (2002). They consider a sample of young aged 19 to 21 between 1995 and 1999 and estimate the probability of attending College conditional on High School completion controlling for unobserved ability, labor market conditions, family background, and proxies for current and permanent parental income. They find that current parental income is an important determinant of College attendance, which they interpret as evidence of binding credit constraints at College, and weak evidence of selection bias. A related contribution is BMM for Brazil. They distinguish between four levels of education and estimate the changes in the ability distribution at each level. They find evidence of sizable compositional changes at Intermediate and College levels. However, the changes in composition can not fully account for the large increase in the College premium.

The "demand-side" literature focuses on the impact of trade liberalization and a series of labor market reforms promoted in Latin America in the 1990s that increased the demand for skilled labor in production. This second branch of literature is vast and counts on many different contributions for each Latin American country.[6] Most empirical studies have found evidence of a technological change that increased the demand for skilled labour, while a smaller set of studies have estimated a positive impact of trade opening on the skill premium.[7] However, when the relative importance of the trade liberalization and of technology explanation has been compared,

[6] Goldberg and Pavcnik (2004) and Winters, McCullach and McKay (2004) provide two exhaustive surveys of the literature.

[7] On the impact of a skill-biased change in production, see, among the others, Attanasio, Goldberg and Pavcnik (2002) for Colombia and Corseuil and Muendler (2003), Pavcnik, Blom and Goldberg (2002) and Holm-Nielsen and Verner (2001) for Brazil, Bustos (2005) for Argentina and Verhoogen (2008) for Mexico. On the impact of opening to trade, see, among the others, Hanson and Harrison (1999) for Mexico and Lisboa, Menezes-Filho and Schor (2004), Gonzaga, Menezes-Filho and Terra (2006) and Giovannetti and Menezes-Filho (2006) for Brazil.

a SBTC appears as the driving force behind the increase in the skill premium.

Behrman, Birdsall and Szekely (2000) have performed an overall evaluation of the major market reforms that characterized LACs in the 1980s and 1990s. They construct six indexes for six main policy reforms and use a panel data set on eighteen LACs for the period between 1980 and 1998 to estimate the impact of the reforms on the relative wages to Higher versus Primary and to Higher versus Intermediate Education.[8] They find that trade liberalization did not have any significant impact on the changes in the wage differentials while the return to Higher Education significantly increased when the economies became more technologically advanced and the exports of high-technology products increased.

Avalos and Savvides (2003) find that increased trade openness is related to the reduction in wage differentials between high and low educated, while exposure to foreign direct investment, greater imports of machinery and equipment and R&D transfer from developed economies are associated with increases in wage differentials.[9]

The analysis developed in this paper is also related to the literature that uses dynamic general equilibrium (GE) models to study wage inequality. The closest contributions are HLT and Lee and Wolpin (2006).[10]

HLT is the first paper to have quantified the importance of the GE effects to explain the changes in wage inequality. They develop and simulate a GE model of training on the job, investment in education and savings that they use to explain the increase in the skill premium observed in the US between the 1970s and the 1990s. They model SBTC as a permanent technological shift toward skilled labor in production and find that the increase in wage inequality can be explained as a result of skill-specific supply responses to a SBTC.

As in HLT, I jointly model savings and educational choices and I characterize SBTC as a permanent shift toward skilled labour in production. However, my model differs in at least three important aspects. First and most importantly, I do allow for credit constraints that are assumed away in HLT framework. Second, I develop an OLG model of parents and children where parents can use financial bequests and education as two alternative strategies to increase child lifetime utility. This creates important interactions between education and saving investments, while HLT do solve for an individual problem and savings are exogenously redistributed to the new

[8] The six reforms they consider are: privatization of former state enterprises, trade liberalization, capital account liberalization, domestic financial market liberalization, tax reforms and labor market reforms.

[9] In practice trade opening and skilled-biased technological change are very related since a change in production towards skilled labour could be the effect of increased exposure to international markets.

[10] There are also a number of contributions that study the importance of the general equilibrium effects on the distribution of education and inequality in steady state. The closer contribution is Gallipoli, Meghir and Violante (2007).

born at the end of life with no direct links between saving choices of successive generations. Finally, I do not model on-the-job investment.

Lee and Wolpin (2006) (LW - hereafter) develop a structural dynamic GE model of work, schooling and sector-specific occupational choice to explain the main facts that characterized the U.S. labor market in the 1970s, 1980s and 1990s. One important fact is the rise in the College wage premium. The structural estimation of the model allows them to explain the evolution of the skill premium over time. They find that SBTC was the main determinant of the increase in the premium.

The model I develop is also an attempt to evaluate alternative explanations of the increase in wage inequality. However, it differs from LW in at least one important dimension: I do model saving choices, while LW assume that individuals can not save. The absence of savings in LW framework does not allow assessing the impact of credit constraints and risk aversion on changes in wage inequality, which in my model will be crucial to understand the evolution of inequality. Also, because of the absence of saving decisions, LW take physical capital to be exogenous. In the model I develop, since I assume that the economy is open to the international financial markets, physical capital does not have a direct role in the determination of the equilibrium prices. However, it does have an important indirect effect, since it has an impact on education choices and therefore on the amount of the different types of human capital aggregates in equilibrium.

A recent contribution that develops a GE model of savings and educational choices with credit constraints and risk is Gallipoli, Meghir and Violante (2007) (GMV - hereafter). The model of GMV has a richer structure than the one developed in this paper, the main difference being that it includes endogenous labor supply. Also, GMV use their model to address a very different question. They are mainly interested in studying the long run effects of policy interventions on educational choices and the distribution of earnings. The change in wage inequality associated with a particular policy is an outcome measure to assess the performance of the intervention.

The remainder of the paper is organized as follows. Section 2 presents some empirical evidence on changes in wage inequality in Latin America and in Mexico. Section 3 presents the theoretical model. Section 4 defines and characterizes the equilibrium. Section 5 and 6 discuss the solution and the parametrization of the model. Section 7 presents the main results from the simulations. Section 8 gives some concluding remarks. Appendix A describes the data, appendix B gives details of the solution methods.

2 Wage Inequality and Returns' Convexification

In the decade of the 1990s Latin American countries have been characterized by high and increasing relative wages at Higher Education and a falling wage differential between Intermediate and Basic Education.

Table 1*: Difference in log hourly real wage.

Year	1987	2002	Growth
BRAZIL			
Higher vs. Intermediate	0.828	1.100	**33%**
Intermediate vs. Basic	0.927	0.557	**-40%**
COLOMBIA**			
Higher vs. Intermediate	0.832	1.092	**31%**
Intermediate vs. Basic	0.551	0.429	**-22%**
MEXICO			
Higher vs. Intermediate	0.375	0.646	**72%**
Intermediate vs. Basic	0.451	0.384	**-15%**

*All adult population aged between 25 and 60.

**Data for Colombia refer to 1986 and 1998.

Table 1 reports the difference between the mean log hourly real wage at Higher and Intermediate and at Intermediate and Basic Education for three main LACs. The values have been computed by using the National Household Survey of each of the three countries and considering the adult population aged between 25 and 60.

The premium to Higher Education is substantial: in 2002 it is over one hundred per cent in Brazil and Colombia and around sixty-five per cent in Mexico.[11] In addition, it is rising over time, with an increase of around thirty per cent in Brazil and Colombia and of over seventy per cent in Mexico.At the same time Intermediate graduates have lost ground. From a value of around ninety-two per cent in Brazil and fifty-five per cent in Colombia and Mexico at the

[11] The value of the premium to Higher education is much higher than the corresponding figure for developed countries. For example, Carneiro, Heckman and Vytlacil (2005) find a value for the College premium in the US of around 15-20 per cent.

end of the 1980s, relative wages at Intermediate with respect to Basic Education decreased by around forty per cent in Brazil and twenty-two and fifteen per cent in Colombia and Mexico by the year 2002. As a result of the increase in the relative wages at Higher Education and the decline at the Intermediate level the wage profile has become convex.[12]

Figure 1

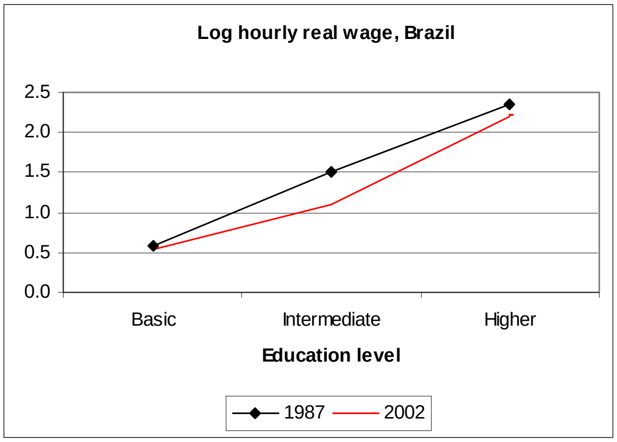


Figure 2

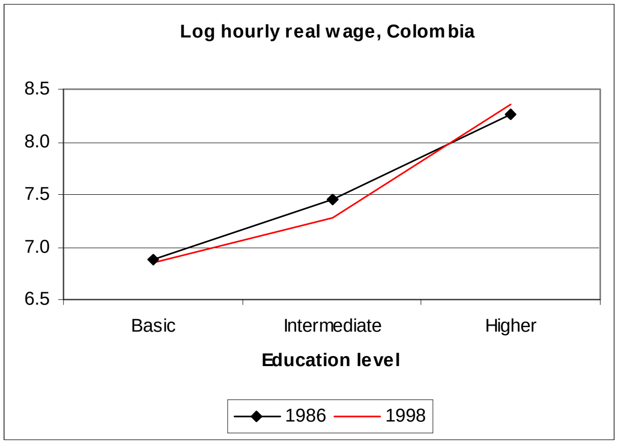
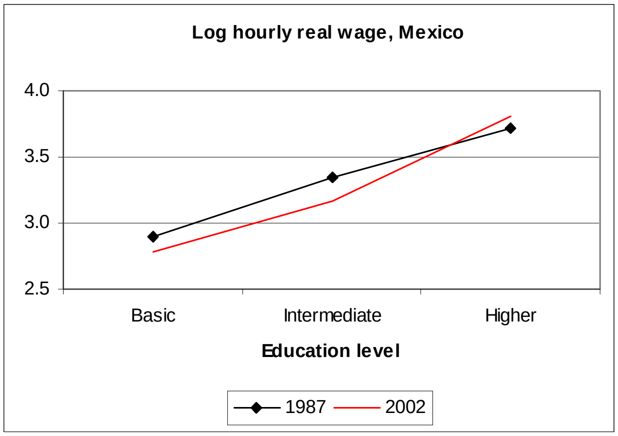


Figure 3



Figures 1 to 3 present the mean residual of the log hourly real wage from a regression that control for gender and age effects in Brazil, Colombia and Mexico at the end of the 1980s and in 2002.[13] The graphs have been obtained by considering the adult population aged between 25

[12]The convexity of the wage function with respect to the level of education changes the typical concave pattern observed in the 1970s and 1980s for LACs (see Patrinos and Psacharopoulos (2004)).

[13]For each of the three countries the difference in the absolute value of the real wages by education between the two years is highly significant. A test of the increase of the relative wage at Higher Education and the decline of the relative wage at Intermediate Education also returns highly significant results.

and 60. In each of the three countries the convexification is apparent: from a linear relationship at the end of the 1980s, wages have become a convex function of the level of education.

In principle, the wage profile could have become convex if both the relative wages at Higher and at Intermediate Education had increased with the relative increase at Higher Education bigger in magnitude than the one at Intermediate. On the contrary, the feature that characterizes the convexification of the wage profile in LACs is the decrease in the relative wages at Intermediate with respect to Basic Education due to the drop in the mean wage at the Intermediate level. In Colombia and Mexico the reduction of the wage at Intermediate came together with an increase in the mean wage at Higher Education. On the contrary, in Brazil the mean wage at Higher Education was lower at the end than at the beginning of the 1990s. However, the relative wage at Higher Education increased also in this country since the decrease in the wage at Higher Education was smaller in magnitude than the one at Intermediate.

2.1 Returns' convexification in Mexico

In the rest of the paper I focus on Mexico. Mexico offers an interesting setting to study the convexification for two main reasons. The first reason is that there are available panel data on wages, so that it is possible to obtain an estimate of the log wages net of unobserved ability for a first assessment of the importance of the changes in the composition on unobservables to produce the convexification.

The data come from the Mexican Employment Survey, ENEU (*Encuesta Nacional de Empleo Urbano*), collected by the Mexican national statistical office. I consider all the years between 1987 and 2002. A description of the Survey together with details on the construction of the education groups is presented in Appendix A. The ENEU is a quarterly household Survey representative of the Mexican urban population and collects wage information on every working individual at least twelve years old over five consecutive quarters. Using the observations on wages over the four consecutive quarters of a given year, a simple fixed effects regression can be used to estimate the log wages net of unobserved ability by level of education. The details of the estimation procedure are described in Section 6.4.

Figure 4

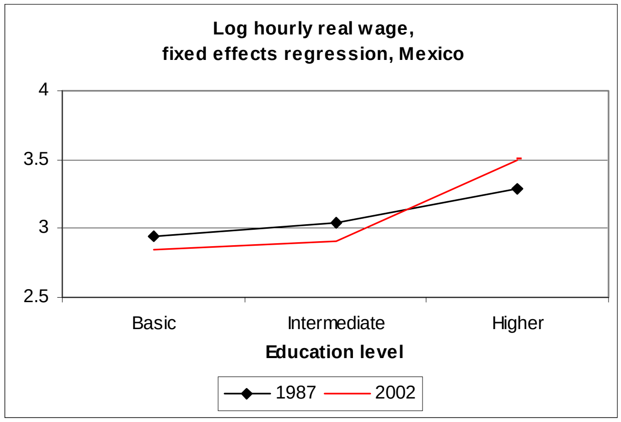


Figure 4 presents the mean log hourly real wages from the fixed effects regression for the year 1987 and 2002. The convexification is even more evident than in Figure 3 and it is once more driven by the drop in the remuneration at Intermediate Education. The results in figure 4 provide preliminary evidence that the changes in the ability composition by level of education did not drive the convexification. The general equilibrium model that I will develop and simulate will be used for a rigorous assessment of the role of ability to produce the convex wage profile.

The change in the shape of the wage profile is driven by the drop in the remuneration at Intermediate Education. There was also a drop in the wage at Basic Education but of a smaller magnitude. The salaries of workers with low education at the bottom of the wage distribution are typically the most affected by changes in labor market institutions such as the level of minimum wage and the degree of unionization that characterize an economy. If the minimum wage had become more binding or the power of the unions had become much stronger, the wage at Basic Education could have been prevented from falling too much, resulting into a decrease of the wage gap between Intermediate and Basic Education. However, the empirical evidence for Mexico shows that in the 1990s minimum wages became less binding[14] and the unions significantly lost power[15].

The second reason why Mexico offers an interesting setting to study the convexification is that during the 1990s this country underwent a series of reforms that changed the structure of production and made the economy more open to foreign investment. The reform effort culminated in 1994 when Mexico became a member of the Organization for Economic Cooperation

[14] See Bosch and Manacorda (2007).

[15] See Fairris (2000).

and Development (OECD) and entered the North American Free Trade Agreement (NAFTA) with the US and Canada. In the same year Mexico was hit by a severe financial crisis, the "Peso crisis", which resulted into a massive devaluation of the national domestic currency. Between 1994 and 1996 Mexican GDP decreased by seven per cent a year. The recovery from the crisis was rather quick and by the end of 1995 Mexico had already reentered the international capital markets.

The reform effort and the opening to foreign investment resulted into an increase in the use of skilled labor and in the production of skill-intensive goods. As discussed in paragraph 1.1, several contributions have found that NAFTA and the economic reforms of the 1994-1996 period increased the demand for skilled labor in production. More recently, Verhoogen (2008) find the Peso crisis rather than the opening to trade being associated with an increase in the use of skilled labour and in the production of high-quality goods. The "quality-upgrading" mechanism that he proposes is able to explain the increased demand for skilled workers that is observed within rather than between plants.

Disregarding the main explanation of the increased skill-intensity, all the papers agree that in the 1990s Mexican economy underwent a structural change towards the use of skilled labor in production.[16] I will estimate the yearly relative increase in the demand for skilled labor and I will use the general equilibrium model for a quantitative assessment of its role to produce the convexification.

The increased demand for skill was not met by a proportional increase in its supply. Figure 5 presents the log relative wages by education for each year between 1987 and 2002. The relative returns to Higher Education follow an upper sloping trend with respect to both Intermediate and Basic Education. On the contrary, the relative returns to Intermediate versus Basic Education decline.

Figure 6 presents the evolution of the relative supply by education. From the end of the 1980s there has been a significant progress towards skill upgrading with a steadily increasing trend of the relative supply at both Intermediate and Higher Education. The upward-sloping trend of the relative supply at Intermediate is consistent with the decreasing returns at this level. On the contrary, the increase in the relative returns to Higher Education is evidence of a demand for skilled workers that more than outweighs the supply increase for this group.[17]

[16] In the 1990s the share of skilled workers increased in all sectors and the exports of skill-intensive goods steeply rose.

[17] Figure 6 has been obtained by aggregating over all adult Mexican workers in each year of the sample between 1987 and 2002. Any individual that migrated from Mexico between this two years would not be in the sample. Mexico is the largest source of migration to the US and the immigration from Mexico to the US increased exponentially in the 1990s. Therefore,

Figure 5

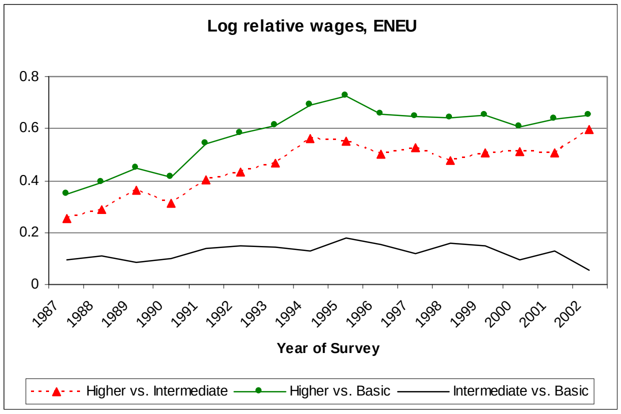
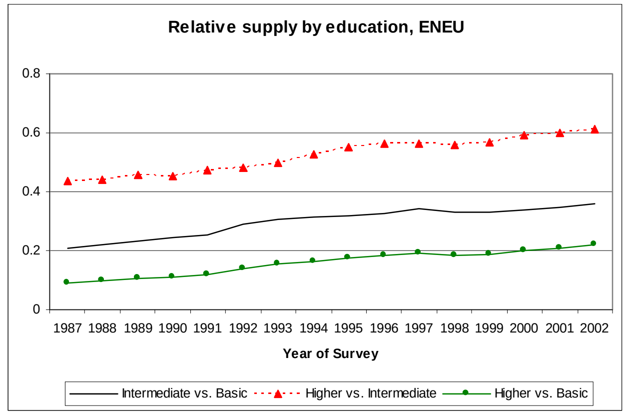


Figure 6



3 The model

The economy starts at some initial year t. In each year it consists of overlapping generations of parents and children that live together for four periods: a pre-school period and three periods necessary to complete all three education levels from Basic up to Higher Education. At $t = 1$ each cohort schooling and wealth distribution are taken as exogenous initial conditions. From $t = 2$ these distributions evolve endogenously as a result of parental maximizing behavior.

3.1 Supply Side: Household Decision Problem

A continuum of individuals is born at each time t. Each individual lives for eight periods, four as a child and four as a parent. As a child the individual lives with the parent that works full time and maximizes utility which is a function of joint household consumption. In the first two periods consumption is the only choice variable. In the third and in the fourth period child education becomes an additional choice variable: together with how much to consume, the parent chooses each period whether to send to child to school or to work.

In the first period the child is in pre-school age, in the second period is sent to compulsory Basic Education. In the third and fourth period the child can be sent either to school or to work. If the child is sent to school, the parent has to pay a fixed cost that is education-specific. If sent to work, the child works full time and gives her earnings to the parent. At the end of the fourth period the parent dies and leaves a bequest of financial assets to the child. The child

migration could potentially be an important factor behind the changes in the relative returns to schooling. However, the Mexican migration was entirely of workers with Basic Education. Therefore, it would not be able to explain the double change in the wage differentials that characterizes the convexification.

starts the adult life with the level of education completed during childhood and an amount of assets given by parental bequest.

Labor supply is perfectly inelastic and the wages clear the labor markets. The wage of an individual i with education level j in period t is given by:

$$w^i_{j,t} = p_{j,t} * \exp(e^i_{j,t}) \qquad j = 1, 2, 3 \tag{1}$$

with

$$e^i_{j,t} = \eta^i + g_j(age_{it}) + z^i_{j,t} \tag{2}$$

where $p_{j,t}$ is the equilibrium price for education level j at time t and $e^i_{j,t}$ denotes labor efficiency of individual i, which is a function of the individual's ability endownment, η^i, $g_j(age)$ is an education-specific polynomial in age which reflects the growth of wages with experience, and $z_{j,t}$, an education-specific i.i.d. uninsurable shock that is received at the beginning of period t and is assumed to be normally distributed with mean $\mu_{z_{j,t}}$ and variance $\sigma^2_{z_{j,t}}$.

The individual's ability endownment, η^i, represents the permanent component of human capital. It is a measure of ability and all unobservable family background factors that have a permanent impact on human capital formation. The ability endownment is assumed to be perfectly transmitted between successive generations: each individual inherits at birth the ability endowment of her parent.

It is assumed that individuals correctly anticipate all current and future skill rental prices, p. Then, given the vector of current and future skill prices forecasted from age a onwards, $p(a)$, the ability endownment, own and child education and the amount of financial assets at age a, parental maximization problem is given by:

$$V^{\eta}_a(j^P, j^C_a, A_a, p(a)) = \max_{\{c_a, I_a\}_{a=\underline{a}}^{a=\overline{a}}} E\left\{\sum_{a=\underline{a}}^{\overline{a}} \beta^{a-\underline{a}} U\left(\frac{c_a^{1-\gamma}}{1-\gamma}\right) + \beta^3 \lambda V^{\eta}_{\underline{a}}(j^P_{\underline{a}}, A_{\underline{a}}, p(\underline{a}))\right\} \tag{3}$$

$$A_{a+1} = A_a(1+r) + w_{jP,a} + [(1-I_a)w_{jC,a} - I_a F_{jC}] - c_a \tag{4}$$

$$A_a \geq -B_a \qquad \forall\ a = \underline{a}, ..., \overline{a} \tag{5}$$

$$A_a \geq 0 \qquad a = \overline{a} \tag{6}$$

where λ is the degree of parental altruism, $\underline{a}$ ($\overline{a}$) denote the age of the parent at start (end) of the adult life, j^C_a is the education level of the child at age a and j^P denotes the level of education of the parent that is fixed throughout adulthood. $V^{\eta}_{\underline{a}}(j^P_{\underline{a}}, A_{\underline{a}}, p(\underline{a}))$ is the child's lifetime utility once adult with child education, $j_{\underline{a}}^{C}$, normalized to zero since in the first period of adulthood consumption is the only choice variable. The value functions are indexed by η since the ability endownment is received at birth and is perfectly transmitted across generations, thus characterizing the parent-child dynasties over time.

c_a and A_a are, respectively, joint household consumption and financial assets at age a. $w_{jP,a}$ ($w_{jC,a}$) is parental (child) wage given parental (child) education level jP (jC) at age a. I_a is an indicator function taking the value of one when the child is sent to school and zero otherwise. If the child is sent to work, the parent receives child earnings, $w_{jC,a}$; if the child is sent to school, the parent pays the fixed costs, F_{jC}, for the jC schooling level attended by the child. E denotes expectations that reflect uncertainty due to the presence of the uninsurable idiosyncratic shocks to earnings and β is the discount factor.

The utility function is assumed to be strictly increasing and concave in consumption, so that absolute risk aversion is decreasing in individual's wealth, the impact of risk on investment decisions being higher for poorer than for wealthier households.[18]

The optimization problem in (3) is solved under three main constraints. The first constraint is a standard period budget constraint with the term in square brackets switching on when child education becomes a choice variable. The second constraint is a borrowing restriction imposing a limit B_a on the amount of net indebtedness allowed at any age a. The third constraint is a terminal condition that prevents parents from dying in debt: they can not leave debts to their children.

The borrowing limit, B_a, can take any value between zero, which corresponds to the maximum level of credit constraints of no possible borrowing, and an upper bound that is given by the expected lifetime earnings at age a under the worst possible negative shock to earnings z. The upper bound represents the maximum amount that an individual will always be able to repay without violating the no-debt condition specified in equation (6).

3.2 Demand Side: Aggregate Production Function

The representative firm operates a constant returns to scale technology over physical and human capital. I assume there are no adjustment costs for physical and human capital and that there

[18] We assume that the utility function takes a simple CRRA formulation:
$U(c) = \frac{(c)^{1-\gamma}}{(1-\gamma)}$
where γ is the reciprocal of the intertemporal elasticity of substitution.

are no shocks to the aggregate production. The production function in year t is given by:

$$Y_t = Z_t K_t^{\alpha} HH_t^{1-\alpha} \tag{7}$$

where Y_t denotes aggregate output, K_t is aggregate physical capital and HH_t is the aggregate human capital.[19] α denotes the share of physical capital in production which is assumed to be constant over time and Z_t is the technology factor that is normalized to one in all years.[20]

I assume that the economy is small and open to the world financial markets. Capital flows into or out of the country so that the marginal product of physical capital equals the world interest rate, r.[21]

I consider three types of human capital corresponding to the three education levels that the individuals can complete and I specify the aggregate human capital in year t, H_t, as a nested CES function over the three types of human capital, H_1, H_2, H_3, which represent, respectively, the human capital of those completing Basic, Intermediate and Higher Education.

The choice of how to combine the three human capital inputs in the HH aggregate is of crucial importance since it determines the elasticity of substitution (ES) between the H factors, which drives the direction and the magnitude of the general equilibrium effects. The simplest specification that assume the same ES between all human capital pairs implies a direct relationship between an increase in the relative supply and a decline in the relative returns between any education pair, which is not what is observed in the data. As shown in figure 5 and 6, between 1987 and 2002 there was both an increase of the relative supply and of the relative returns to Higher Education with respect to both Intermediate and Basic Education.

A convenient way to allow for a different ES between pairs of human capital is to combine them within a CES specification. Given three human capital inputs, there are three ways of nesting them within a CES aggregate: $\widetilde{HH}_1 = \Gamma_1(H_3, \Gamma_2(H_2, H_1))$, $\widetilde{HH}_2 = \Gamma_2(H_2, \Gamma_2(H_3, H_1))$ and $\widetilde{HH}_3 = \Gamma_3(H_1, \Gamma_2(H_2, H_3))$, where Γ_1, Γ_2 and Γ_3 are CES aggregators.

The CES functional form imposes symmetry restrictions on substitution elasticities. For $\widetilde{HH}_1$, the ES between H_3 and H_1 is restricted to be the same as the one between H_3 and

[19] This specification of the production function assumes that there are no complementarities between physical and human capital. This assumption is motivated by the near-constancy of the share of physical capital in production estimated for LACs (see Bosworth, 1998, Harrison, 1994 and Hoffman, 1993).

[20] Given the assumption of no population growth and the normalization of Z, there is no growth in steady state. Growth will occur only during the transition towards a steady state as a result of the reallocation of efficiency units of labor from less to more productive combinations of the different types of human capital.

[21] The constancy of the world interest rate implies that the economy's capital to labor ratio is fixed over time. Also, this assumtion implies that firms face no credit constraints. Differently from individual households, they can freely borrow in the international capital markets at the fixed rate r. There are no financial intermediaries that can borrow money from firms and lend it to households.

H_2. This restriction contrasts with factor elasticities estimates for LACs suggesting that the ES between Higher and Intermediate Education differs from the one between Higher and Basic Education.[22]

For $\widetilde{HH}_2$, the ES between H_2 and H_3 is restricted to be the same as the one between H_2 and H_1. This specification assumes a complementarity in production between H_3 and H_1 for which there is no empirical support from Mexican data.

For $\widetilde{HH}_3$, the CES restricts the ES between H_1 and H_2 to be the same as the one between H_1 and H_3. This specification assumes the existence of complementarities between Intermediate and Higher Education in production. This assumption finds empirical support in the production structure of the Mexican economy that is characterized by two main sectors: a formal sector of semi and high-skilled workers and an informal sector of low skilled ones. Also, a specification test finds this assumption to be consistent with the data (see section 6.6).

Therefore, HH_t is specified as it follows:

$$HH_t = [(1-\delta_{s,t})H_{u,t}^{\rho} + \delta_{s,t}H_{s,t}^{\rho}]^{\frac{1}{\rho}} \tag{8}$$

where $H_{u,t}$ and $H_{s,t}$ are, respectively, the human capital aggregate for unskilled and skilled labour at time t.

$H_{u,t}$ correspond to $H_{1,t}$ while H_s is given by:

$$H_{s,t} = [(1-\alpha_{3,t})H_{2,t}^{\theta} + \alpha_{3,t}H_{3,t}^{\theta}]^{\frac{1}{\theta}} \tag{9}$$

The time-varying and skill-specific parameters δ and α in equations (8) and (9) denote the shares of the human capital factors in the aggregate production. Changes in δ and α reflect variations in the productivity and in the demand of the different inputs in production. The parameter ρ determines the ES between unskilled and skilled labor, which is given by $ES_{u,s} = ES_{1,2} = ES_{1,3} = \frac{1}{1-\rho}$, while θ determines the ES between Intermediate and Higher Education, which is given by $ES_{2,3} = \frac{1}{1-\theta}$. Complementarities between Intermediate and Higher Education require that $\rho > \theta$. If either ρ or θ is zero, the corresponding nesting is Cobb-Douglas.

The aggregate stock of the jth human capital in year t, $H_{j,t}$, is given by the sum of the efficiency weighted supply of labor of the individuals that completed the jth education level at

[22] See Manacorda, Sanchez-Paramo and Schady (2006).

time t:

$$\begin{aligned} H_{j,t} &= \sum_i h^j_{i,t} \qquad\qquad j = 1,2,3 \\ h^i_{j,t} &= (j^i_t * e^i_{j,t}) \end{aligned} \tag{10}$$

where j^i_t is the supply of labor type j of individual i in year t and $e^i_{j,t}$ is the efficiency index defined in equation (2).

Differently from physical capital, labor is not internationally mobile and its remuneration is set domestically. Under the assumption of perfectly competitive markets and profit maximization by firms, the price for education level j in year t, p_{jt}, is given by the marginal product of the jth aggregate human capital:

$$p_{1,t} = G_t(1-\delta_{s,t})H^{\rho-1}_{1,t} = \frac{\partial Y_t}{\partial H_{1,t}} \tag{11}$$

$$p_{2,t} = G_t(\delta_{s,t})[\alpha_{3,t}H^\theta_{3,t} + (1-\alpha_{3,t})H^\theta_{2,t}]^{\frac{\rho-\theta}{\theta}}(1-\alpha_{3,t})H^{\theta-1}_{2,t} = \frac{\partial Y_t}{\partial H_{2,t}} \tag{12}$$

$$p_{3,t} = G_t(\delta_{s,t})[\alpha_{3,t}H^\theta_{3,t} + (1-\alpha_{3,t})H^\theta_{2,t}]^{\frac{\rho-\theta}{\theta}}\alpha_{3,t}H^{\theta-1}_{3,t} = \frac{\partial Y_t}{\partial H_{3,t}} \tag{13}$$

where $G_t \equiv Y_t(1-\alpha)\frac{1}{[\delta_{s,t}H^\rho_{s,t}+(1-\delta_{s,t})H^\rho_{u,t}]}$.

By taking the ratios of the expressions above, I can derive the expressions for the relative returns to schooling:

$$\frac{p_{2,t}}{p_{1,t}} = \frac{\delta_{s,t}}{(1-\delta_{s,t})}(1-\alpha_{3,t})\left(\frac{H_{1,t}}{H_{2,t}}\right)^{1-\rho}\left\{(1-\alpha_{3,t}) + \alpha_{3,t}\left[\frac{H_{3,t}}{H_{2,t}}\right]^\theta\right\}^{\frac{\rho-\theta}{\theta}} \tag{14}$$

$$\frac{p_{3,t}}{p_{2,t}} = \frac{\alpha_{3,t}}{(1-\alpha_{3,t})}\left(\frac{H_{3,t}}{H_{2,t}}\right)^{\theta-1} \tag{15}$$

$$\frac{p_{3,t}}{p_{1,t}} = \frac{\delta_{s,t}}{(1-\delta_{s,t})}\alpha_{3,t}\left(\frac{H_{1,t}}{H_{3,t}}\right)^{1-\rho}\left\{\alpha_{3,t} + (1-\alpha_{3,t})\left[\frac{H_{2,t}}{H_{3,t}}\right]^\theta\right\}^{\frac{\rho-\theta}{\theta}} \tag{16}$$

where equations (14), (15) and (16) define, respectively, relative returns to Intermediate versus Basic, Higher versus Intermediate and Higher versus Basic Education.

The degree of complementarity between Intermediate and Higher Education is an important determinant of the changes in relative returns. An increase in the amount of human capital at Intermediate level has two different effects: a standard supply effect (SE) and a complementarity effect (CE). The standard SE is clear from the term in round brackets in equation (14) and (15). An increase in H_2 that is bigger in magnitude than the increase in H_3 decreases the relative return to Intermediate with respect to Basic Education and increases the relative return to Higher with respect to Intermediate Education. The CE is given by the term in square brackets in equations (14) and (16). If $\rho > \theta$, that is if Higher and Intermediate Education are more complementary than Higher and Basic (or Intermediate and Basic), with respect to a production with no complementarities, an increase in H_2 further decreases the relative return to Intermediate with respect to Basic Education and increases the relative return to Higher with respect to Basic Education. The results of the estimation of ρ and θ are reported in Section 6.6. The estimates confirm the presence of complementarities between Intermediate and Higher Education.

Activities of the government are not central to the analysis. The government neither collects taxes nor redistributes them.

4 Equilibrium

This section is divided in two parts. The first part defines the equilibrium of the economy. The second part discusses the properties of the equilibrium steady state.

4.1 Definition

Given an initial distribution of ability, financial assets and education and the set of model's parameters, a competitive equilibrium is given by an exogenous interest rate, r, a sequence of vector prices, $p_t = [p_{1,t,}p_{2,t}, p_{3,t}]$, aggregate labor inputs, $H_t = [H_{1,t,}H_{2,t}, H_{3,t}]$, consumption and education choices, $[c_t, I_t]$, for $t = 0, 1, 2, ...$, such that, at any time t:

1. Given the prices $[p_{1,t,}p_{2,t}, p_{3,t}]$, c_t and I_t optimally solve households' maximization problem.
2. Given the prices $[p_{1,t,}p_{2,t}, p_{3,t}]$, firms choose optimally the production factors and prices are marginal productivities:

$$p_{j,t} = \frac{\partial Y_t}{\partial H_{j,t}} \qquad \forall j$$

3. The labor and good markets clear:

$$\sum_i h^j_{i,t} = H_{j,t} \qquad \forall j$$

From Walras' Law, the good market clearing condition is derived residually by integrating over the individual budget constraint.

An equilibrium steady state is a competitive equilibrium with stationary prices and distributions, that is an equilibrium such that $[p_t, H_t] = [p, H]$ for all t.

4.2 Properties of the steady state

Proposition 1. *If the utility function $U(.)$ belongs to the space of bounded functions with the sup norm and is strictly concave and continuously differentiable, an equilibrium steady state exists. The equilibrium steady state depends on the initial conditions that characterize the model economy which are given by the distribution of education, wealth and ability and the set of model's parameters. Therefore, the steady state is history-dependent.*

Discussion. The existence of the steady state is guaranteed by the properties of the utility function. The solution to each of the functional equations that characterize the parental decision problem can be found by applying the Contraction Mapping Theorem and verifying that the Blackwell sufficient conditions are satisfied.[23] These conditions are always satisfied for the class of bounded functions with the sup norm, to which the CRRA utility function belongs to.

The non-uniqueness result has been proved for the case of two education groups by Mookherjee and Ray (2003). The proof can be restated for any finite number of skill groups.

The result is due to the presence of a limited number of education levels that are available investment options under credit constraints. Each *jth* education level is characterized by a fixed cost that the parent has to be able to pay to pursue the investment. Credit market imperfections do not allow free borrowing and restrict the number of feasible investment opportunities. Families can not optimally fine-tune their educational investments that depend on the amount of resources they have and on the level of the credit constraints. Therefore, the steady state the economy converges to depends on the initial distribution of cash in hand and on the set of model's parameters.

Uniqueness is restored when there are perfect capital markets or a continuum of education levels with a corresponding continuum stream of fixed costs associated to them. In these situations, for any initial distribution of education and financial assets, families can always afford to pay a *jth* fixed cost and therefore invest into the corresponding *jth* education level. Mookherjee and Ray (2003, 2007) give a formal proof of how uniqueness is restored in a model with credit constraints and a continuum of occupation groups, provided that the interest rate is kept exogenous.

Proposition 2. *There is no steady state with all the population either skilled or unskilled.*

[23] See Stokey and Lucas (1989).

The intuition behind this result lies in the endogenous nature of the skill prices. If the entire population was either skilled or unskilled, the increase in the skilled (unskilled) human capital would result into a decrease of the corresponding price, which would give incentives to invest in the unskilled (skilled) human capital.

Proof. Suppose that all the population was unskilled with all individuals starting to work after completion of Basic Education. Then, $H_u \to \infty$. From the conditions for the equilibrium prices, it follows that $p_u \to 0$ and $p_s \to \infty$.

Condition (1) of the equilibrium definition states that in equilibrium education choices maximize utility, which means that the expected value of sending the child to work after completion of Basic Education must be higher than the expected value of sending the child to Intermediate Education, that is:

$$U(A(1+r)+w_{jP}+w_{jC=1})+\beta E\max\_Work > U(A(1+r)+w_{jP}-F_2)+\beta E\max\_Sch$$

where $w_{jC=1}$ is the unskilled wage that the child earns if she is sent to work and F_2 is the fixed cost the parent has to pay in order to send the child to Intermediate Education. $E\max\_Work$ and $E\max\_Sch$ denote, respectively, the maximum between the conditional value functions of sending the child to school and to work in the next period given the choice of sending the child to work and to school in the current period. Given that $p_s \to \infty$, $E\max\_Sch \to \infty$. Therefore, the above inequality can not hold, leading to a violation of the condition that defines the equilibrium of the economy and therefore to a contradiction.

Corollary. *With respect to the initial distribution of skills, the distribution of skilled and unskilled human capital at steady state is characterized by either zero mobility with no positive measure of individuals switching between skilled and unskilled human capital or by mobility in both directions.*

Discussion. The no-mobility scenario would always arise in the absence of any randomness in the model, that is if the idiosyncratic shocks to wages were removed and the dynasties made optimal choices in a perfectly certain world. The strict concavity of the utility function and the absence of perfect credit markets imply that the richer the family the smaller is the utility sacrifice in educating the child, so the bigger the incentive to send her to the highest level of schooling. Subsequent generations will have the same set of incentives and the steady state will be characterized by a positive correlation between the amount of cash in hand and of education: richer (poor) families with high (low) levels of schooling. A formal proof is provided by Moohkerjee and Ray (2003) that develop a model where there are no random components.

In the presence of random shocks, a full-mobility scenario becomes possible. For a sufficiently low level of the fixed costs of schooling, the steady state will be characterized by mobility in both direction. A formal proof is provided by Owen and Weil (1998) that consider an economy where the source of randomness comes from an ability endowment that is modelled as a random shock independently drawn for each generation.

Proposition 3. *The aggregate supply of human capital of skill j in period t, $H_{j,t}$, is a non-decreasing function of the jth skill premium but it may be non-monotonic with respect to the other skill premiums.*

Proof. Consider the aggregate supply of human capital of skill level j, H_j. Consider first an increase of size $\varepsilon > 0$ in the jth skill price p_j. Any parent that chooses to invest in child education at p_j, will do the same at $p_j + \varepsilon$.

Therefore H_j will be a non-decreasing function of p_j.

Consider now an increase of size $\varepsilon > 0$ in p_j, $j = j+1, j+2$. In this case, the response of the aggregate supply H_j is ambiguous. As an example, let us consider the effect of an increase in p_3, the premium to Higher Education, on the aggregate supply of Intermediate human capital, H_2. The increase in p_3 will give incentives to invest at all levels of schooling.

In particular, both the supply of Intermediate and of Higher Education are expected to increase. However, whether the proportion of Intermediate graduates increases or decreases will depend on the net difference between the "new comers", i.e. individuals going from Basic to Intermediate Education, and the "leavers", i.e. individuals going from Intermediate to Higher Education. Therefore, H_2 may be a non-monotonic function of p_3.

5 Model's solution

In any given year t for all cohorts of parents and children the household's problem is solved recursively by backwards induction. Consider a parent aged a in year t. Denote with x_a and $V_a^{\eta}(x_a)$, respectively, the vector of state variables and the optimum value function at age a. η denotes the ability endownment received at birth. x_a includes the vector of all current and expected skill prices, the level of parental and child education and the amount of financial assets.

In the first two periods of adult life consumption is the only choice variable. The parent chooses consumption to solve the problem:

$$V_a^\eta(x_a) = \max_{c_a} \left\{ U(c_a) + \beta E_{z_{a+1}} V_{a+1}^\eta(x_{a+1}) \right\} \tag{17}$$

subject to the constraints (2), (3) and (4) defined above. Consumption is properly weighted by an equivalence scale to account for the presence of the child. Expectations are taken over the idiosyncratic shock to earnings z.

In the third and fourth periods child education becomes a second choice variable. I first compute the optimal consumption choices by solving two separate maximization problems conditional on having sent the child to school and to work. Then, I compute the education choices by comparing the conditional value function of sending the child to school and to work: the child is sent to school whenever the expected value of investing in schooling is at least as high as the expected value of sending her to work.

A complete description of the solution method is provided in the first section of Appendix B.

Starting from a set of initial conditions defined by the model's parameters and the distribution of education, wealth, ability and of the idiosyncratic shocks to earnings, the model is solved in any year of the transition towards the steady state. The method for solving for the equilibrium price sequence that clears the market in every period of the transition is described in the second section of Appendix B.

6 Parametrization

This section discusses how I parametrize the model.

6.1 Initial distribution wealth and education

I set the initial distribution of education using the Mexican Employment Survey (ENEU) in 1987. I divide the workers in two categories: the adult population that is made up by all heads of households aged between 25 and 60 with Basic, Intermediate and Higher Education and the population of young living with their parents and aged between 15 and 24 with completed Basic and Intermediate Education. I use the mean proportions by education in the adult population to set the initial education distribution of the parents and the mean proportions of the young with Basic and Intermediate Education for the education distribution of the children in the third and fourth periods. In the pre-school period all children have by definition zero education and they all complete compulsory Basic education.

The ENEU does not record information on wealth. I use instead the Mexican Expenditure

Survey, ENIGH (*Encuesta Nacional de Ingresos y Gastos de los Hogares*), which is available in 1984, 1987 and every two years since then and contains detailed information on individuals' consumption and assets. I set the initial wealth distribution to a lognormal with mean and standard deviation from the distribution of financial assets of workers aged between 25 and 60 in 1992.[24]

6.2 Preferences and costs of schooling

The coefficient of relative risk aversion, γ, is set to 0.9, which gives a value of around 1.1 for the elasticity of intertemporal substitution (EIS). The value is taken from Arrau and Wijnbergen (1991) that estimate for Mexico a value for the EIS between a lower bound of 0.8 and an upper bound of 1.4. The limit on net indebtedness, B, is set to zero, which corresponds to the maximum level of credit constraints. The consumption is adjusted to account for the presence of the child. I use an equivalence scale equal to 0.7 for a child reflecting the average calories intake of a child relative to an adult as reported by the Mexican National Nutritional Institute.[25]

I set the values of the fixed costs of schooling, F_j, for each jth education level so that the model matches the education distribution of the workers aged between 26 and 60 in the ENEU in 1987. I find $F_1 = 0.035$, $F_2 = 0.26$ and $F_3 = 0.64$, which implies that the costs at Intermediate level are around seven times the ones at Primary and the costs at Higher Education are around eighteen times the ones at Primary.[26]

Assuming that parent-child dynasties are linked by fully altruistic preferences, the altruism parameter, λ, is set to one.

6.3 Interest rate and capital share

The value of the real interest rate, r, is set to a US benchmark value. I choose a value of five per cent, which is the mean long term real interest rate on the US Treasury Bills published by the US Treasury Department for the 1990s. Given an average working life of the adult Mexican

[24] The ENIGH reached its final structure only in 1992 with significant changes in the questionnaire and data collection in the years before. For this reason, I parametrize the wealth distribution using the first and second moments in 1992 instead of the moments in 1987. The results of the simulations are not sensitive to this choice.

[25] See Hernández, Chávez and Bourges, 1987.

[26] The F_j intend to measure the total direct costs of education, which include the costs of school material and maintanance. An empirical counterpart of these costs is provided by the nationally representative Mexican Family House Survey (MxFLS), which collects high quality data on a rich set of variables for a cross section of Mexican households in 2002. The Survey contains a detailed set of questions on education costs and distinguishes between tuition fees, the costs of exams, books, school material, uniforms and the maintenance costs for public and private schools. Summing over the different categories and computing the mean for public and private education, the costs of Intermediate education are around eight times the ones at Primary while the ones at Higher education are around nineteen times the ones at Primary.

population of approximately thirty years, the model period is set to seven years. Therefore, the interest rate in the model is $r = (1.05^7 - 1) \cong 0.41$. Setting the yearly discount factor equal to the inverse of $(1 + r)$, the discount factor in the model is $\beta = 1.05^{-7} \cong 0.71$.

The capital share, α, is set equal to 0.35, which is the average value between the lower and the upper bound that has been estimated in empirical studies that use data from LACs.[27]

6.4 Labor efficiency

The human capital aggregates are defined as the sum of the efficiency weighted labor supplies of all individuals with a given education level. The jth labor input is the product of the number of workers with the jth education level and the efficiency index defined in equation (2).

In order to be able to simulate the model, I need to obtain an estimate of the mean and variance of the distribution of unobserved heterogeneity, η, and of the idiosyncratic shock, z, and an estimate of the coefficients of the education specific age polynomials, g_j.

To this aim, I exploit the panel data feature of the Mexican Employment Survey (ENEU) over four consecutive quarters. As described in Appendix A, the ENEU is a quarterly household Survey representative of the Mexican urban population and it collects detailed employment and wage information on every working individual at least twelve years old over five consecutive quarters, the four quarters of a given year and the first of the following year.

I specify an education-specific log wage equation for individual i with education level j in quarter qr:

$$\ln w^i_{j,qr} = p_{j,qr} + g_j(age_{iqr}) + u^i_{j,qr} \qquad j = 1, 2, 3 \tag{18}$$

$$u^i_{j,qr} = \eta^i + z^i_{j,qr} \tag{19}$$

$$z_{j,qr} \sim N(0, \sigma^2_{z_{j,qr}}) \tag{20}$$

where $\ln w^i_{j,qr}$ is the real log hourly wage of individual i with skill level j in quarter qr. $p_{j,qr}$ is the (log) price for education level j, η^i is a permanent individual-specific effect, $g_j(.)$ is a skill-specific quadratic polynomial in age and $z_{j,qr}$ is an education specific i.i.d. shock received by the individual in quarter qr.

I construct panels of individuals by matching workers by the position in an identified household, number of years of education and age. I consider all wage workers between the age of 15

[27] See Bosworth (1998) for a discussion of the empirical issues involved in the estimation of the capital share in Mexico and Harrison (1994) and Hoffman (1993) for two cross-countries empirical studies that use a capital share that varies between the value of 0.3 and 0.4 for a group of LACs.

and 60 and I follow them for the four successive quarters in a given year.[28]

For each year of the sample between 1987 and 2002 I run the following fixed effects regression:

$$(\ln w^i_{j,qr} - \ln \overline{w}_{ji}) = (\ln w_{j,qr} - \ln \overline{w}) + g_j(age_{i,qr}) - \overline{g_j(age_{i,qr})} + (u^i_{j,qr} - \overline{u}_{ji}) \qquad (21)$$

where the upper-bar variables denote time averages over the four quarters in year t for the ith individual with the jth education level. $g(.)$ is a polynomial of order two and the term $(\ln w_{j,qr} - \ln \overline{w})$ is modelled as time dummies.

For the purposes of the simulation I require the unconditional distribution of ability as reflected by the fixed effect η^i. I use the estimate:

$$\widehat{\eta}_i = \frac{\sum_{t=1}^{T(i)} \ln w^i_{qr} - \widehat{\ln w_{qr}} - g(\widehat{age_{i,qr}})}{T(i)} \qquad (22)$$

where $T(i)$ is the total number of observation available on individual i. The estimated fixed-effects give an estimate of the $\{\eta^i\}$ distribution over the working population.[29]

Figure 7

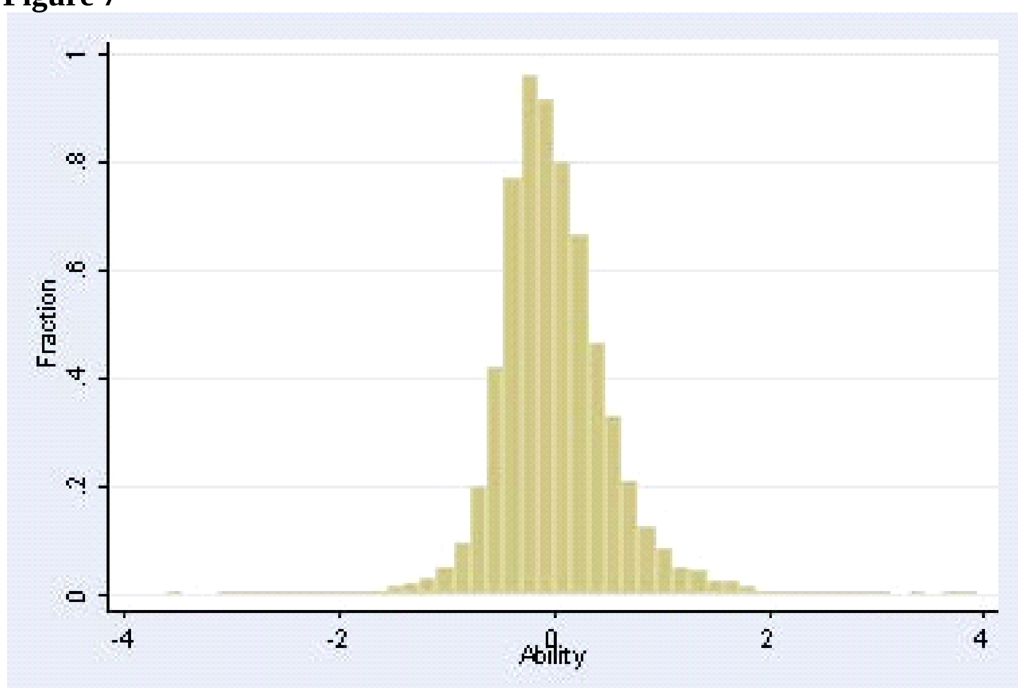


Figure 7 presents the empirical frequencies of $\widehat{\eta}_i$ obtained for the year 1987. The first and

[28] I consider all wage earners that are aged between 15, which is the minimum legal working age and 60, which is the average retirement age in Mexico.

[29] Effectively, since I run yearly fixed effects regressions, the distribution of the fixed effects is identified by the individuals that are still in education, that is by the individuals aged between 15 and 25.

second moment of the distribution of $\widehat{\eta}_i$ are used to parametrize the distribution of ability in the model. I find $\sigma_\eta^2 = 0.25$ and $\mu_\eta = 0$.[30]

Given the estimation of equation (21), I can treat as observable the following:

$$u^i_{j,qr} = \ln w^i_{j,qr} - g_j(age_{i,qr}) - \ln w_{j,qr} - \eta^i \tag{23}$$

I use the residuals from the wage equation to obtain an estimate of the distribution of the idiosyncratic shock. I assume that $u^i_{j,qr}$ is a normally distributed i.i.d. shock with mean zero and variance $\sigma^2_{u_j}$. I use the second moment of the distribution of $u^i_{j,qr}$ for each education group to parametrize the distribution of z. I find $\sigma^2_{u_1} = 0.0625$, $\sigma^2_{u_2} = 0.0676$, $\sigma^2_{u_3} = 0.0784$.

Finally, the coefficients of the quadratic polynomials $g_j(.)$ provide the estimates of the education-specific experience effects.

6.5 Human capital aggregates

Following HLT, I identify the human capital aggregates, $H_{j,t}$, by combining the data on the wage bills paid in each year to the different education groups with the time series of the skill prices estimated for each year between 1987 and 2002.

Define the wage bill $WB_{j,t}$ as the total earning payments received by the individuals of a given education group in each year. Then:

$$WB_{j,t} = \widehat{p}_{j,t} * H_{j,t} \qquad j = 1,2,3 \tag{24}$$

where $\widehat{p}_{j,t}$ is the estimated market price of workers with education level j in year t.

Therefore:

$$H_{j,t} = \frac{WB_{j,t}}{\widehat{p}_{j,t}} \qquad j = 1,2,3 \tag{25}$$

Setting to one the value of each of the $\widehat{p}_j$ in 1987, it is possible to identify the series of the utilizied human capital stocks normalized to 1987.

In order to compute the wage bills I need a data set that is representative of the entire Mexican population. The ENEU collects information on urban areas only so it can not be used to this purpose. I use instead the ENIGH, that is nationally-representative and reports individual earnings together with detailed information on assets and consumption.

For each year and education group I compute the wage bill summing over the individual wages of all primary wage earners between the age of 15 and 60. I linearly interpolate the

[30] The distribution of ability is taken to be time-invariant. I therefore ignore any possible heterogeneity in terms of ability endownments between successive cohorts active in the labour market in different years.

available data for the missing years.

The total number of workers and the wage bills by education for each year between 1987 and 2002 are reported in figures 8 and 9. The drop of the wage bills between 1994 and 1996 is the result of the Peso crisis that hit Mexico in 1994: the total wage bill declined in real terms by around 29 per cent at Basic and by around 35 and 48 per cent at Intermediate and Higher Education. Wages started increasing again steadily for all education levels from the end of the 1990s.

Figure 8

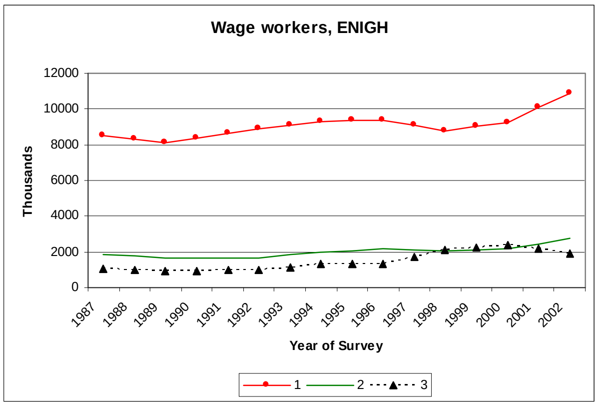
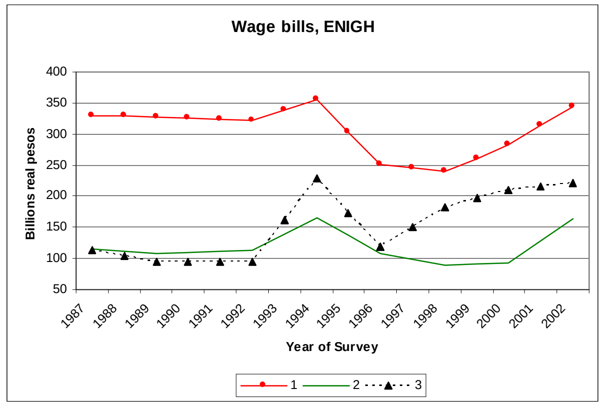


Figure 9



In order to estimate the $\widehat{p}_{j,t}$ I run the fixed effect regression described in equation (21) for each year of the sample between 1987 and 2002 and I compute the predicted mean log hourly real wage for each education level net of unobserved ability. Given the wage bills and the skill prices, I divide the wage bills by the exponentiated value of the skill prices to obtain the time series of the human capital aggregates for each year and education group.

The $\widehat{p}_{j,t}$ in equation (25) represent the prices to education level j in year t after controlling for unobserved ability. HLT assume that at older ages changes in wages are solely due to changes in skill prices and to depreciation and do not depend any more on ability. Therefore, as an estimate for the $\widehat{p}_{j,t}$ they simply use the mean wage by education and year in the US CEX. On the contrary, I use of the estimated $\widehat{p}_{j,t}$ net of the unobserved ability, which makes the identification of the H factors consistent with the ability distribution estimated from the data and used to simulate the model.[31]

Figures 10 and 11 present the series of the estimated skill prices and human capital stocks normalized to 1987. As in the aggregate wage bills, the impact of the Peso crisis does also shows

[31] Using the same procedure as HLT does not change the main result of the estimation of the production funtion.

up in the skill prices: between 1994 and 1996 the log skill price decreased by around 12 per cent at Basic and by around 10 per cent at Intermediate and Higher Education.

All the three human capital aggregates show a positive growth in the 1990s but with a rather non-monotic trend. In particular, the growth of H_3 dropped sharply between 1994 and 1996 and the one of H_2 became positive only from the year 2000. The decrease (increase) in the value of the estimated H is due to an increase (decrease) in the level of the estimated prices of the corresponding H factor which grew proportionally more (less) than the total remuneration of the factor.

Figure 10

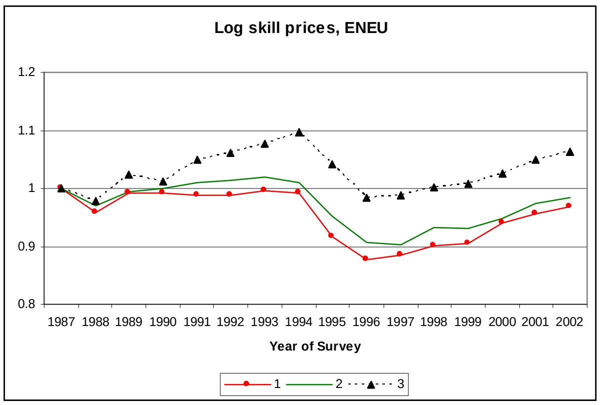
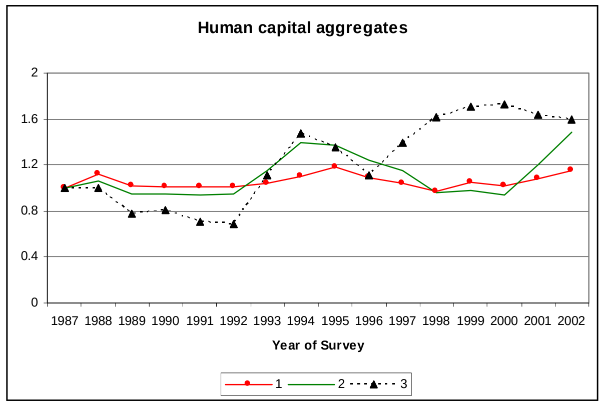


Figure 11



6.6 Aggregate technology

The parameters of the production function can be estimated by using the expressions for the returns to schooling in equations (14) to (16).

Log linearizing equation (15), I obtain:

$$(\log p_{3,t} - \log p_{2,t}) = [\log \alpha_{3,t} - \log(1 - \alpha_{3,t})] + (\theta - 1)(\log H_{3,t} - \log H_{2,t}) \tag{26}$$

where $\log \alpha_{j,t}$ denotes the time series of the relative demand shifts for skill level j measured in log quantity units.

Rewriting the above expression in terms of wage bills I obtain:

$$(\log WB_{3,t} - \log WB_{2,t}) = [\log \alpha_{3,t} - \log(1 - \alpha_{3,t})] + \theta(\log H_{3,t} - \log H_{2,t}) \tag{27}$$

Equation (27) can be used to obtain a direct estimate of the values of the relative demand shifts and the elasticity of substitution between Higher and Intermediate Education.

The time-varying factor shares $\alpha_{2,t}$ and $\alpha_{3,t}$ reflect changes in the productivity and in the demand of the different human capital factors in the aggregate production. A commonly used specification in the literature assumes that the logs of the factor shares follow a simple trend-stationary process:

$$\log \alpha_{j,t} = \phi_{0,j} + \phi_{1,j} * t + e_{j,t} \tag{28}$$

where $\phi_{0,j}$ is a skill-specific constant, t denotes a linear time trend and e_t is a normally distributed i.i.d. shock at time t for skill level j.

Combining equation (27) and (28), the value of the parameter determining the elasticity of substitution between Higher and Intermediate Education, θ, and the logs of the factor share can be estimated from a regression of the ratio of log wage bills on the ratio of human capital aggregates, a linear trend and a constant. In order to correct for a possible endogeneity bias I apply an IV estimator using as instrument the first lag of the difference of the logs of the human capital factors.[32]

Then, given unskilled human capital corresponds to Basic Education and skilled human capital is a composite of Higher and Intermediate human capital, I can estimate a regression of the ratio of log wage bills for skilled and unskilled on the ratio of skilled and unskilled human capital, a linear trend and a constant to obtain an estimate of ρ and of the logs of the factor share δ_s.

Table 2* : Estimation of the production function

	Higher vs. Intermediate^	Skilled vs. Unskilled'
Difference log H	0.772***	0.857***
	(0.114)	(0.127)
Time trend	0.026***	0.016***
	(0.007)	(0.005)
Implied ES	4.4	6.9

* Standard errors in parentheses.
^ Dependent variable: difference log wage bill Higher and Intermediate.
' Dependent variable: difference log wage bill Skilled and Unskilled.

Table 2 presents the estimates obtained for Higher versus Intermediate Education and for Skilled versus Unskilled. The estimates of ρ and θ are consistent with the presence of complementarities between Intermediate and Higher Education: $\rho > \theta$. The value for the ES between

[32] Given the spike in the wage bills and the skill skill prices in 1994 (see figures 8 and 9), I tried alternative specifications that allow the trend to vary for the pre and post 1994 period. However, the interaction term never turns out to be significant.

Higher and Intermediate (Skilled and Unskilled) is of around 4.4 (6.9).[33]

The production function assumes that the ES between H_1 and H_2 is the same as the one between H_1 and H_3. I use a joint estimation procedure to test for this assumption. From the conditions for the equilibrium prices I derive an equation for the relative return to Intermediate and to Higher with respect to Basic Education and I apply a log-linearization to obtain two equations for the difference in the log wage bills. Then, I jointly estimate this system of equations and I test for the equality of the coefficients of the human capital terms. The test gives a value of chi-squared of 0.09 with a P-value of 0.7703. Therefore, it is unable to reject the null hypothesis.

By simple substitution of equation (28) in (27), it is easy to see that the coefficient of the time trend is the difference of the trend for Higher and Intermediate ($\phi_{1,3} - \phi_{1,2}$) and for skilled and unskilled ($\phi_{1,s} - \phi_{1,u}$), which give an estimate of the yearly relative demand-shift of Higher with respect to Intermediate and skilled with respect to unskilled labour. I call the coefficient of the time trend for skilled and unskilled the "skill-bias" parameter. I will use it to model SBTC as an annual increase in the demand of skilled labour in production.[34]

The wage bill equations can also be used to identify the values of the shares parameters in 1987, $\alpha_{3,1987}$ and $\delta_{s,1987}$, which will be used as the baseline estimate to run the model's simulations.

Let us consider the identification of $\alpha_{3,1987}$. Given equations (27) and (28) and the fact that $\alpha_{2,1987} = (1 - \alpha_{3,1987})$, I have that $\log\left[\frac{\alpha_{3,1987}}{(1-\alpha_{3,1987})}\right] = (\beta_0 + \beta_1 * 1987)$, where $\beta_0 = (\phi_{0,3} - \phi_{0,2})$ and $\beta_1 = (\phi_{1,3} - \phi_{1,2})$. Therefore, $\alpha_{3,1987} = \frac{\exp(\beta_0+\beta_1*1987)}{(1+\exp(\beta_0+\beta_1*1987))}$. Using the wage bill equation for skilled and unskilled human capital, the equivalent of equation (27) for $\delta_{s,1987}$ and the definition of the unskilled labor share as one minus the skilled share, I can use the same steps to identify $\delta_{s,1987}$. Following this procedure, I obtain a baseline estimate of 0.55 for α_3 and of 0.686 for δ_s.

7 Simulations

In this section I use the model for a quantitative assessment of the role of an increased demand for skill, credit constraints and uninsurable risk to produce the convexification of the wage profile.

[33] Manacorda, Sanchez-Paramo and Schady (2006) use a cross section of LACs that includes Argentina, Brazil, Chile, Colombia and Mexico in the 1980s and 1990s and find a value of around 4.5 for the ES between Higher and Intermediate education and a value of around 2.5 for the ES between Skilled and Unskilled. Therefore, they find a lower ES between skilled and unskilled than between Higher and Intermediate. They allow for a different ES between age groups and they pool five LACs together. The difference with my results could be due to any of these two features.

[34] The estimate of the time trend for Higher with respect to Intermediate education is close in magnitude to the one estimated by Manacorda, Sanchez-Paramo and Schady (2006) for a cross-section of Latin American countries and to the one estimated by HLT and Katz and Murphy (1992) on US data.

First, I define a baseline economy as the one with the share of skilled labor estimated at the end of the 1980s and the highest level of credit constraints of no possible borrowing. Second, I assess the role of an increased demand for skill by simulating the model in the presence of a SBTC in partial and general equilibrium, and with and without the complementarity between Intermediate and Higher Education. Third, I assess the role of the "supply factors" by simulating the model for different levels of credit constraints and uninsurable risk. I compute the transitional dynamics and the final steady state, which is characterized by the amount of the three aggregate labor inputs and the value of the corresponding equilibrium prices. To highlight the main results, I only focus on the steady state.[35]

7.1 Skill-biased technological change

A SBTC is modelled as a permanent increase of δ_s, which measures the contribution of skilled labor to the production of the aggregate output. An increase in its value is to be interpreted as an increase in the productivity or in the demand of this type of labor. The size of the increase is given by the "skill-bias" parameter obtained from the estimation of the production function in section 6.6. As reported in table 2, I estimate a skill-bias parameter of 0.016. The estimated value of the factor share δ_s for the year 1987 is used as the baseline case that defines the no-SBTC scenario. Then, starting from this value, I assume that δ_s increases linearly by two per cent a year for five consecutive years and it remains constant at the increased value from the sixth year onwards. Shifts of longer and shorter duration produce qualitatively similar results.

I define the baseline economy as the one with no-SBTC and the maximum level of credit constraints of no possible borrowing. I assess the impact of a SBTC in two consecutive steps. First, I compute the equilibrium prices that the model would predict given the increase in δ_s in the absence of any general equilibrium effect, that is by keeping the supply of each education level fixed at the value in 1987. I call this counterfactual Scenario I. Then, I define a second counterfactual where the supply is allowed to change in reaction to changes in the prices. I call this counterfactual Scenario II.

[35] All the results on the transitional dynamics are available from the author upon request.

Figure 12

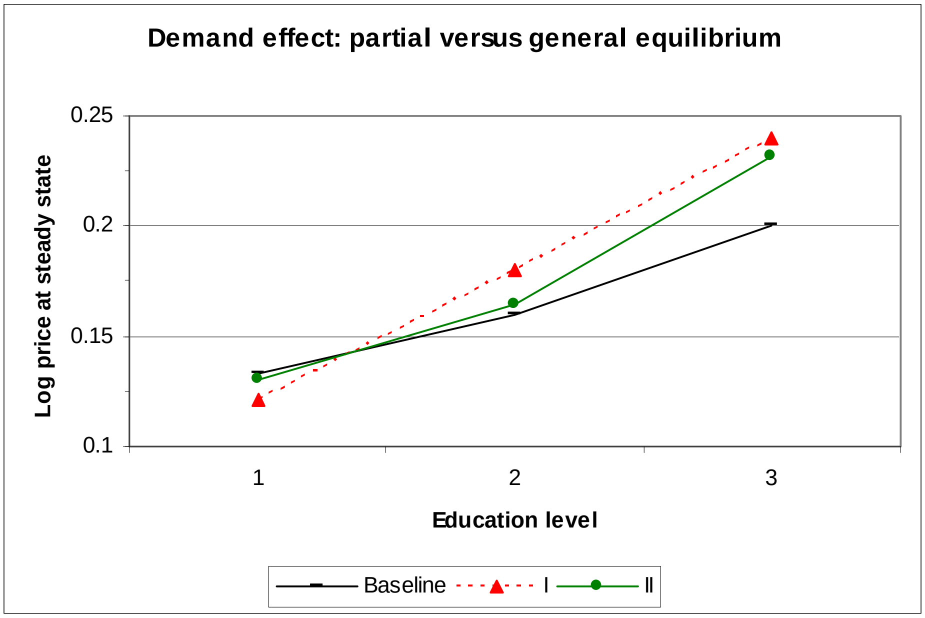


Figure 12 presents the equilibrium log prices at steady state in the baseline case and in Scenario I and II. At baseline, in the absence of any SBTC, the model predicts a linear relationship between the mean log prices and the level of education.

Once the share of skilled labor is increased, Scenario I produces a steeper prices' schedule. As clear from equations (11) to (13) and the fact that $\delta_u = (1 - \delta_s)$, an increase in δ_s decreases the equilibrium price at Basic and increases the prices at both Intermediate and Higher Education. With respect to the baseline, both the relative returns to Higher with respect to Intermediate and to Intermediate with respect to Basic Education increase.

Scenario II allows the supply to react to the changes in the prices. The difference with respect to scenario I is due to the supply reaction to a SBTC. The increase in the prices at Intermediate and Higher Education gives incentives to invest after compulsory schooling. However, the supply at Intermediate increases more than at Higher level, so the price at Intermediate decreases more and the steady state price schedule becomes convex.

The complementarity between Intermediate and Higher Education is of fundamental importance to this result. Since I estimate a value of ρ that is higher than the value of θ, which means that Higher and Intermediate Education are more complementary than Higher and Basic (or Intermediate and Basic), in addition to the standard supply effect, an increase in H_2 that is bigger in magnitude than the increase in H_3 further decreases the relative return to Intermediate with respect to Basic Education and increases the relative return to Higher with respect to Basic Education.

Figure 13

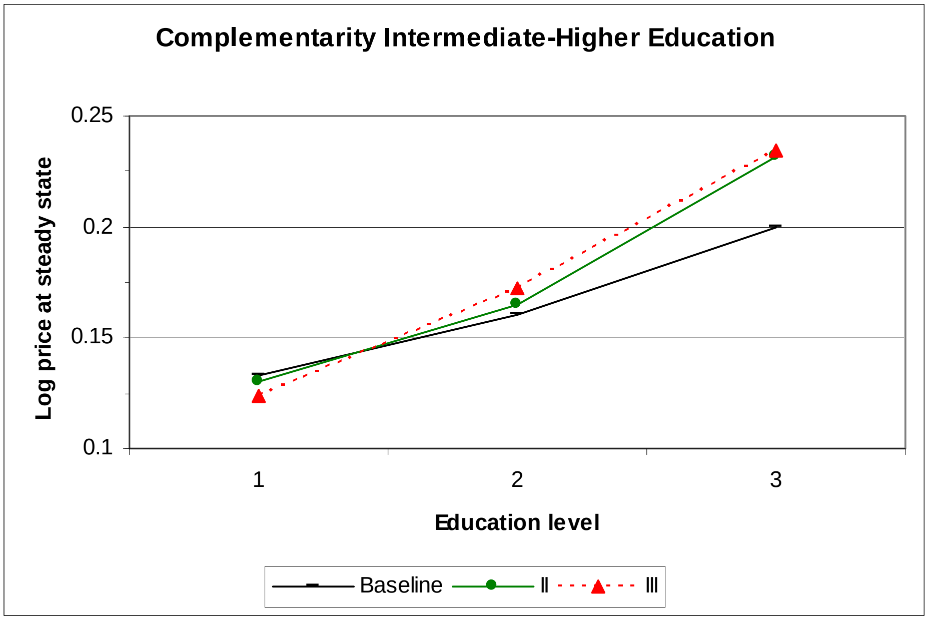


Figure 13 quantifies the importance of the complementarities between Intermediate and Higher Education. I define a third counterfactual, Scenario III, that assumes that the ES between skilled and unskilled labor input is the same as the one between Intermediate and Higher Education. As it is clear from the graph, in the absence of any complementarity between Intermediate and Higher Education, the equilibrium wage function in steady state is linear.

Figure 12 and 13 show the importance of the general equilibrium supply effects and of the ES between aggregate labor inputs to produce the convexification. Yet, scenario III is far from the convexification observed in the data: both the relative returns to Higher and to Intermediate Education are increasing and the price at Intermediate lies above the one at baseline.

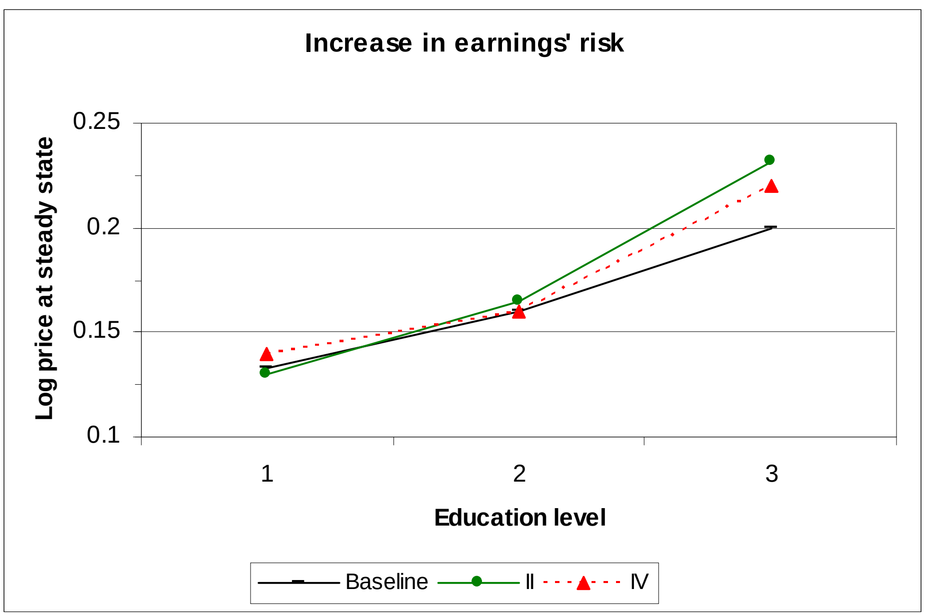
All the resuts up to now have been obtained for a given level of uninsurable risk and the maximum level of credit constraints of no possible borrowing. Both an increase in the level of risk and in the amount that is possible to borrow would change the stock of available resources and therefore the incentives to invest in education. In the following paragraphs I investigate the role of a change in the level of risk and credit constraints to produce the convexification.

7.2 Uninsurable risk

The level of uninsurable risk has a direct impact on the amount of human capital investment. Positive (negative) shocks can change the fortune of dynasties that are born poor (rich) and allow some families (not) to have enough resources to invest up to Higher Education.

The variance of the distribution of the shocks to earnings is a measure of the extent of risk faced by the households. A higher variance of the shocks increases the level of earnings' volatility. In Section 6.4 I estimated the variance of the shock by education for each year between 1987 and 2002. For each education group, I take the highest estimated value of the variance and I use these estimates to define a fourth counterfactual, Scenario IV, that is given by Scenario II with the new variances of the shocks. From the values of 0.0625, 0.0676 and 0.0784 for the variance of the shock to Basic up to Higher Education used in Scenario II, the values are increased to 0.0778, 0.0936 and 0.1246 for each of the three education levels. Figure 14 reports the steady state prices' schedule at baseline and in Scenario II and IV.

Figure 14



The possibility of receiving big positive shocks that could allow financing education up to the highest level increases the option value of education as a profitable investment opportunity and gives incentives to invest in education after compulsory schooling. Therefore, the supply at Basic declines and the one at Intermediate increases with a corresponding increase in the price of Basic and a decrease in the price of Intermediate Education.

Scenario IV starts showing the first feature of the convexification: the relative return to Higher Education increases and the relative return to Intermediate Education declines. However, a high level of risk also means that there is a positive probability of receiving big negative shocks: the increase in the supply at Intermediate is not big enough to produce the sizable drop in the price at this level, which is the characterizing feature of the convexification.

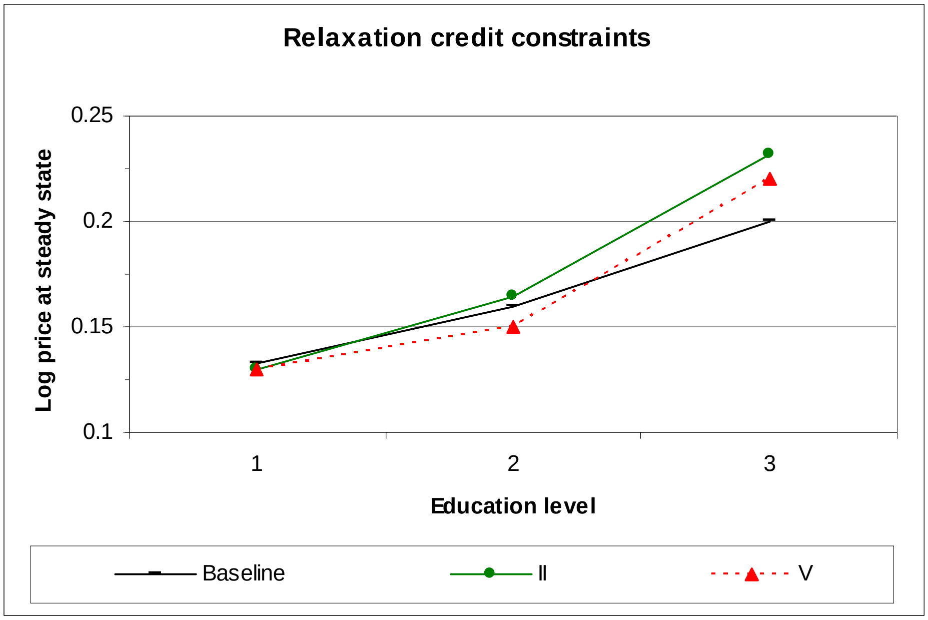
The results up to now have been obtained under the assumption of no possible borrowing. In this situation, only households that start out with enough resources can fully react to the increase in the return to education. I now investigate how the results change when borrowing is allowed.

7.3 Credit constraints

The assumption of no possible borrowing significantly restricts the set of feasible educational investments and the extent of upward mobility from Basic to Higher Education. I am not aware of any estimate of the actual degree of credit market imperfections in Mexico, so there is no empirical benchmark to fix the value of B, the maximum amount that is possible to borrow. The internal consistency of the model allows to set the upper and lower bound of B. The lower bound can not fall below zero, which defines the no-borrowing case considered so far. The upper bound is fixed at the present discounted value of lifetime earnings under the worst possible idiosyncratic negative shock, which is the maximum amount the parents will always be able to repay without violating the no-debt condition stated in equation (6).

I define a fifth counterfactual, Scenario V, that is given by Scenario II with a relaxation of the credit constraints to the upper bound of the values that B can take. Figures 15 reports the wage profile at steady state in the baseline and in Scenario II and V.

Figure 15



The possibility of borrowing allows more people to complete Intermediate and Higher Education, so investment in human capital at both levels increases and therefore the prices decrease. However, the supply increase at Intermediate level does not translate into a proportional increase at Higher Education, so the price at Intermediate decreases more than at the Higher level. With respect to the baseline, the relative return to Higher versus Intermediate increases and the relative return to Intermediate versus Basic Education decreases with the price at Intermediate at a level that falls below the value at baseline. There is a positive relationship between the borrowing limit and the size of the supply increase at Intermediate: the more it is possible to borrow, the higher is the investment in education after compulsory schooling. Unreported results show that there is a borrowing treshold below which the size of the supply increase at Intermediate is not enough to produce a decrease in the equilibrium price at this level so that the price falls below the value at baseline.

Scenario V presents all the features that characterize the convexification in Mexico**.** Tables 3 and 4 present the growth of the level and relative log education prices in the ENEU and in each of the five model scenarios with respect to the baseline case. Only in the fifth scenario the price at Intermediate falls below the value at baseline. Also, the size of the decline is very close to the one estimated in the data. In terms of relative price, the model predicts an increase of around 75 per cent of the relative return to Higher with respect to Intermediate Education and a decline of around 26 per cent of the relative return to Intermediate versus Basic Education. They are close in magnitude to the ones estimated from the data.

Table 3: Growth of the level of log education prices with respect to baseline*

Log price education	ENEU (1987-2002)	I	II	III	IV	V
Basic	-3%	-9%	-2%	-7%	5%	-2%
Intermediate	-5%	13%	3%	7%	0%	-6%
Higher	6%	20%	16%	17%	10%	10%

*The values of the log prices at baseline are 0.133 for Basic, 0.16 for Intermediate and 0.2 for Higher education.

Table 4: Growth of the relative log education prices with respect to baseline*

Log relative prices	ENEU (1987-2002)	I	II	III	IV	V
Higher versus Intermediate	72%	50%	69%	50%	200%	75%
Higher versus Basic	25%	78%	52%	19%	139%	34%
Intermediate versus Basic	-15%	119%	28%	-26%	48%	-26%

*The values of the relative prices at baseline are 0.04 for Higher-Intermediate, 0.067 for Higher-Basic and 0.027 for Intermediate-Basic.

The convexification is the result of a differential supply reaction by type of labor to a SBTC. Then, it is the combination of a relaxation of the credit constraints and the presence of complementarities between aggregate labor inputs that produced the type of convexification observed in Mexico in the 1990s. The complementarities between Intermediate and Higher Education are responsible for the differential change in the wage gap between Intermediate and Basic and Higher and Intermediate Education, while the relaxation of the credit constraints is necessary to obtain the drop in the level of the remuneration at Intermediate.

As discussed in paragraph 2.1, there is extensive evidence of a SBTC in Mexico. At the same time, there is also evidence of an increased availability of credit in this country. The decade of the 1990s was characterized by a process of financial liberalization and deregulation of the securities markets, which resulted into an increase in the availability of credit to the private sector. As documented by Bandiera, Caprio, Honohan and Schiantarelli (2000), two common measures of monetary depth as the M2 aggregate and the ratio of the total credit to the private sector as a percentage of the gross national GDP both steeply increased in the first half of the 1990s.

The change in the wage profile from linear to convex corresponds to significant changes in the ability distribution by education. Table 5 presents the mean and standard deviation of the ability level by education at Baseline and in Scenario V. The supply increase at Intermediate results into a sharp decline in the mean ability at this level an an increase in the mean ability at Basic Education. Due to the entrance of low-ability individuals together with the high-ability ones, mean ability does also decline at Higher Education. However, the reduction in mean ability at Intermediate is of a much bigger magnitude than the one at Higher Education. Therefore, with respect to the baseline case, the ability gap between Higher and Intermediate Education increases sharply.

Table 5: Ability level by education group

Ability by education	Baseline		Scenario V	
	mean	sd	mean	sd
Ab_BE	-0.226	0.396	0.311	0.276
Ab_IE	0.396	0.314	-0.191	0.436
Ab_HE	0.615	0.283	0.358	0.536

The model can be used for a quantitative assessment of the role of changes in the ability distribution by education to produce the convex wage profile. We can simulate the economy at constant skill prices and compute the changes in the ability distribution that were needed to produce the convexification.

A useful benchmark of the changes in the ability distribution by level of education in a Latin American country over the 1990s is provided by the estimates obtained by BMM for Brazil. Unreported results show that the changes in the mean and variance of ability by education group that were necessary to produce the convexification in the absence of any general equilibrium effects would be more than five order of magnitudes bigger than the ones estimated by BMM. Therefore, changes in the composition of ability by education do not drive the convexification: they are one result rather than the main determinant of it.

8 Conclusion

This paper studies the central feature that characterizes the changes in wage inequality in Latin America in the 1990s: the combination of an increase in the wage differential between Higher and Intermediate Education and a decline in the differential between Intermediate and Basic Education driven by a drop in the real wage at Intermediate. From being linear at the beginning of the 1990s, by the end of the decade the relationship between log wages and the level of education became convex. This has important implications for the process of human capital accumulation. From one side, the non-linearity of the wage function changes the opportunity costs of education, from the other, it reflects changes in its supply. Yet, the empirical literature on LACs so far has taken the supply of education as given and focused on the increase in the wage differential between Higher and Intermediate Education rather than on the convexification.

Using micro-data from Mexico between 1987 and 2002 I show that the convexification is not the result of a composition effect of higher ability into Higher Education but rather of changes in the prices of schooling in a period of economic reforms that many empirical studies have associated with an increased demand for skilled labour in production. I develop, solve and simulate a general equilibrium model of savings and investment in education under credit constraints and uninsurable earnings' risk to evaluate the relative importance of an increased demand for skilled labor with respect to changes in the level of credit constraints and risk to produce the convex profile. The production function is of crucial importance. I specify and estimate a production function that allows for complementarities between Intermediate and Higher Education. I find that Intermediate is more complementary with Higher than with Basic Education.

The results of the simulations show that the general equilibrium effects of changes in the supply of education on its returns are a fundamental determinant of the evolution of wage inequality. Starting from the linear relationship between the level of education and its remuneration at the end of the 1980s, two are the main determinants of the convexification.

On the one hand, the combination of an increase in the relative return to Higher and a decline in the relative return to Intermediate Education is due to the increase in the relative supply at Intermediate in response to an increase in the demand for skilled labour. The complementarity between Intermediate and Higher Education is driving this result. The increased demand for skill increases the return at Intermediate and Higher Education and gives incentives to invest after compulsory schooling. Since Higher Education is more complementary with Intermediate than is Basic Education, the increased supply at Intermediate increases the relative return to Higher with respect to Basic Education and further decreases the relative return to Intermediate versus Basic Education.

On the other hand, the drop in the price at Intermediate level and therefore the extent of the convexification crucially depends on the level of credit constraints. There is a borrowing limit below which the increase in the supply at Intermediate is not big enough to produce the sizable drop in the price at this level.

The findings of this paper suggest that the general equilibrium effects that large changes in the supply of some skills might have on their prices can significantly affect the incentives to invest in education and are a key determinant of the observed changes in wage inequality.

This paper is part of a broad research agenda to study wage inequality in a general equilibrium framework. The convexification of the wage profile seems to have characterized the evolution of wage changes in other developing countries outside Latin America, such as Vietnam in East Asia[36], as well as in some developed countries, such as the US.[37]More generally, the growing literature documenting an increase in wage inequality driven by rising remunerations to the top percentiles of the earnings' distribution suggest that an underlying convexification of the returns to skills could be the driving feature of the recent changes in wage inequality.[38]

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A Appendix - Data

This appendix describes the data set used to estimate the wage process and the construction of the education groups.

A.1 ENEU

The ENEU (Encuesta Nacional de Empleo Urbano) is the Mexican national employment survey collected yearly by Mexican national statistical office, INEGI. It is a quarterly household survey with a rotating panel structure: households are interviewed for five successive quarters and in each quarter 20 per cent of the households are replaced by new households that are interviewed for the first time. The Survey started in 1981 with progressive increase of the geographic coverage. The sample is selected to be geographically and socio economically representative of the national Mexican urban population: by the end of the 1990s the Survey covered approximately 62 per cent of the national urban population and 92 per cent of the cities with population greater than 100,000.

The main questionnaire is divided in three parts, the first with socio demographic and employment information on individuals at least 12 years old, the second with socio demographic data on individuals younger than 12 and the third with information on the characteristics of the house of residence. The employment information is very detailed with several questions on individuals' occupation status, type and characteristics of employment, sector of main and secondary job, contract type, number of working hours, monthly wages, unemployment status and duration and social security taxes paid by the worker's employer in the private and public sector.

A.2 Construction of the education groups

The Mexican education system consists of three main cycles: Primary, Secondary (Lower and Higher) and Post-secondary (University/technical institutes or more). Primary education lasts six years, Lower and Higher Secondary three years each, University four to five years and graduate education two to fouur years (two years are necessary to obtain a Master degree and two additional years to obtain a PhD).

Primary education starts at age 6 and it has always been compulsory. In 1993 Lower Secondary has also became compulsory. At the beginning of the 1990s attainment rates at Primary were above 90 per cent in all Mexico, while attainment rates at Lower Secondary were above 80 per cent in urban areas and below 40 per cent in rural areas. The impact of the policy change on schooling attainment mainly affected rural areas with a large increase in the construction of schools and corresponding increasing attainment rates at Lower Secondary in these areas. Focusing on urban data only motivates the choice of grouping into compulsory education both Primary and Lower Secondary.

Secondary education includes an "academic" and a "vocational" branch that paves the way, respectively, to University versus non-University education. Post-secondary education comprises Universities, 4-years technical institutes and 2-years technical institutes. By far the majority of students are enrolled in University and a very small proportion is enrolled in 2-years technical institutes.

In order to construct the three education groups used in the model, the schooling levels have been aggregated as it follows. The "Basic Education" group includes all individuals that have up to uncompleted Lower Secondary education, the "Intermediate Education" group includes all individuals that have completed Lower Secondary and have up to uncompleted University education and the "Higher Education" group includes individuals who have completed University or more. As in Manacorda, Sanchez-Paramo and Schady (2006), I aggregate the academic and vocational branch into Secondary education considering in the "Intermediate" group all individuals that have completed any of the two branches.

B Appendix - Solution methods

This appendix is divided in two parts. The first part describes the method used to solve the household problem in any given year t. The second part discusses the method used to compute the equilibrium transition towards the steady state.

B.1 Household problem

The solution method described in this section is used to solve the household problem in any given year t.

Denote with $x\left(a\right)$ the vector of state variables at age a, which includes the vector of all current and future skill prices from age a onwards, $p(a)$, the level of adult education, j^P, that is fixed throughout adulthood, the level of child education, j_a^C and the amount of assets at age a, A_a. Then $x(a) = (j_a^P, j_a^C, A_a, p(a))$, with j_a^C, normalized to zero when consumption is the only choice variable.

In order to simplify the notation, I omit the superscript η from the value functions and the subscript t from the value functions, the choice and state variables. For a given vector of prices, the solution method can be summarized in the following steps.

Step 1. Solve the optimization problem in the last period of work life before retirement ($a = \overline{a}$).

Define with $V^{Sch}_{\overline{a}}(j^P, j^C_{\overline{a}}, A_{\overline{a}}, p(\overline{a}))$ and with $V^{Work}_{\overline{a}}(j^P, j^C_{\overline{a}}, A_{\overline{a}}, p(\overline{a}))$, respectively, the conditional value function of sending the child to school and to work and denote with $W^{Sch}_{\underline{a}}(j^P_{\underline{a}}, A_{\underline{a}}, p(\underline{a}))$ and $W^{Work}_{\underline{a}}(j^P_{\underline{a}}, A_{\underline{a}}, p(\underline{a}))$ the initial guess for child lifetime utility as an adult conditional on having sent the child, respectively, to school and to work in the last period of coresidence. $\underline{a}$ denotes the age of the parent in the first period of adult life.

Given $W^{Sch}_{\underline{a}}(.)$ and $W^{Work}_{\underline{a}}(.)$, $V^{Sch}_{\overline{a}}(.)$ and $V^{Work}_{\overline{a}}(.)$ take the following expressions:

$$V^{Sch}_{\overline{a}}(j^P, j^C_{\overline{a}}, A_{\overline{a}}, p(\overline{a})) = \max_{c_{\overline{a}}} \left\{ U(c_{\overline{a}}) + \lambda \beta E_{z_{\underline{a}}} W^{Sch}_{\underline{a}}(j^P_{\underline{a}}, A_{\underline{a}}, p(\underline{a})) \right\} \quad (29)$$

$$s.t. \quad c_{\overline{a}} = A_{\overline{a}}(1+r) + w_{jP,\overline{a}} - F_{jC} - A_{\overline{a}+1} \quad (30)$$

$$j^C_{\underline{a}} = (j^C_{\overline{a}} + 1) = j^P_{\underline{a}} \quad (31)$$

$$V^{Work}_{\overline{a}}(j^P, j^C_{\overline{a}}, A_{\overline{a}}, p(\overline{a})) = \max_{c_{\overline{a}}} \left\{ U(c_{\overline{a}}) + \lambda \beta E_{z_{\underline{a}}} W^{Work}_{\underline{a}}(j^P_{\underline{a}}, A_{\underline{a}}, p(\underline{a})) \right\} \quad (32)$$

$$s.t. \quad c_{\overline{a}} = A_{\overline{a}}(1+r) + w_{jP,\overline{a}} + w_{jC,\overline{a}} - A_{\overline{a}+1} \quad (33)$$

$$j^C_{\underline{a}} = j^C_{\overline{a}} = j^P_{\underline{a}} \quad (34)$$

where r is the fixed real interest rate on financial assets, F_{jC} denotes the fixed costs of schooling for child education level j^C and $w_{jP,\overline{a}}$ and $w_{jC,\overline{a}}$ are, respectively, parental and child wage at age $\overline{a}$ given parental (child) education level $jP(jC)$. λ denotes the degree of parental altruism and expectations are taken over next period shock to earnings, z. Equations (31) and (34) describe the evolution of child education that increases by one unit if the child is sent to school. The level of child education at the end of the last period of coresidence defines the (fixed) education level throughout adulthood ($j^P_{\underline{a}} = j^P$). Expectations are taken with respect to the idiosyncratic shock to wages received at age a, z_a

Step 2. Solve the conditional maximization problems in each period until the first period of adult life.

In the first and second periods the child is, respectively, in pre-school age and in compulsory Basic Education. Therefore, consumption is the only choice variable and child education is normalized to zero.

At age $\underline{a}$, in the first period of adult life, the parent solves the following maximization problem:

$$V_{\underline{a}}(j^P, A_{\underline{a}}, p(\underline{a})) = \max_{c_{\underline{a}}} \left\{ U(c_{\underline{a}}) + \beta E_{z_{\underline{a}+1}} V_{\underline{a}+1}(j^P, A_{\underline{a}+1}, p(\underline{a}+1)) \right\} \quad (35)$$

$$s.t. \quad c_{\underline{a}} = A_{\underline{a}}(1+r) + w_{jP,\underline{a}} - A_{\underline{a}+1} \quad (36)$$

In the second period the maximization problem is given by:

$$V_a(j^P, A_a, p(a)) = \max_{c_a} \{U(c_a) + \beta E \max\} \quad (37)$$

$$s.t. \quad c_a = A_a(1+r) + w_{jP,a} - F_1 - A_{a+1} \quad (38)$$

where F_1 denotes the fixed costs of Primary education and the $E\max$ operator defines the expected value over the maximum between the conditional value functions of the schooling and work alternative in the next period, which is the first period of choice between sending the child to school or to work. It takes the following expression:

$$E\max \equiv E_{z_{a+1}} \max[V_{a+1}^{Sch}(j^P, j_a^C = 1, A_{a+1}, p(a+1)), V_{a+1}^{Work}(j^P, j_a^C = 1, A_{a+1}, p(a+1))]$$

where $j_a^C = 1$ denotes completed Basic Education.

When child education becomes a choice variable, the conditional maximization problems read:

$$V_a^{Sch}(j^P, j_a^C, A_a, p(a)) = \max_{c_a} \{U(c_a) + \beta E \max\_Sch\} \quad (39)$$

$$s.t. \quad c_a = A_a(1+r) + w_{jP,a} - F_{jC} - A_{a+1} \quad (40)$$

$$j_{a+1}^C = (j_a^C + 1) \quad (41)$$

$$V_a^{Work}(j^P, j_a^C, A_a, p(a)) = \max_{c_a} \{U(c_a) + \beta E \max\_Work\} \quad (42)$$

$$s.t. \quad c_a = A_a(1+r) + w_{jP,a} + w_{jC,a} - A_{a+1} \quad (43)$$

$$j_{a+1}^C = j_a^C \quad (44)$$

where F_{jC} is the fixed cost of child schooling level jC and the $E\max\_Sch$ and the $E\max\_Work$ operators define the expected value over the maximum between the conditional value functions of the schooling and work alternative in the next period given the decision of sending the child, respectively, to school or to work in the current period.

They take the following expressions:

$$E\max\_Sch \equiv E_{z_{a+1}} \max[V_{a+1}^{Sch}(j^P, j_{a+1}^C = (j_a^C+1), A_{a+1}, p(a+1)), V_{a+1}^{Work}(j^P, j_{a+1}^C = (j_a^C+1), A_{a+1}, p(a+1))]$$

$$E\max\_Work \equiv E_{z_{a+1}} \max[V_{a+1}^{Sch}(j^P, j_{a+1}^C = j_a^C, A_{a+1}, p(a+1)), V_{a+1}^{Work}(j^P, j_{a+1}^C = j_a^C, A_{a+1}, p(a+1))]$$

Step 3. Compute the new initial guesses for $W_{\underline{a}}^{Sch}(.)$ and $W_{\underline{a}}^{Work}(.)$.

The solution of the model in steps one and two provides the complete set of value functions and optimal saving rules for any combination of the state space variables.

I can use the optimal value function in the first period of adulthood, $V_{\underline{a}}$, as a new initial guess for the child lifetime utility. Denoting with $j_{\underline{a}}^{C}$ the level of education of the child at the end of the last period of coresidence, $V_{\underline{a}}(j^{P} = (j_{\underline{a}}^{C} + 1), A_{\underline{a}}, p(\underline{a}))$ provides the new initial guess for $W_{\underline{a}}^{Sch}(.)$ and $V_{\underline{a}}(j^{P} = j_{\underline{a}}^{C}, A_{\underline{a}}, p(\underline{a}))$ provides the new initial guess for $W_{\underline{a}}^{Work}(.)$. Given the new initial guesses for $W_{\underline{a}}^{Sch}(.)$ and $W_{\underline{a}}^{Work}(.)$, I repeat steps one and two.

Given the conditional value functions for the work and schooling alternative, the child is sent to school when the expected value of investing in schooling is at least as high as the expected value of sending the child to work, that is when the following condition holds:

$$V_{a}^{Sch}(j^{P}, j_{a}^{C}, A_{a}, p(a)) \geqslant V_{a}^{Work}(j^{P}, j_{a}^{C}, A_{a}, p(a)) \quad \forall \quad a = a\_ed, ...\overline{a} \tag{45}$$

where a_ed denotes parental age when child education becomes a choice variable.

Step 4. Repeat steps one to three until the following two conditions are satisfied:

$$||V_{\underline{a}}^{Sch\_Iter}(j^{P}, A_{\underline{a}}, p(\underline{a})) - V_{\underline{a}}^{Sch\_(Iter-1)}(j^{P}, A_{\underline{a}}, p(\underline{a}))|| \leq \varepsilon \tag{46}$$

$$||V_{\underline{a}}^{Work\_Iter}(j^{P}, A_{\underline{a}}, p(\underline{a})) - V_{\underline{a}}^{Work\_(Iter-1)}(j^{P}, A_{\underline{a}}, p(\underline{a}))|| \leq \varepsilon \tag{47}$$

where ε is an arbitrarily small number and $||.||$ denotes the distance between the conditional value functions in the first period of adulthood in two consecutive iterations.

B.2 Transitional dynamics

This section describes the method used to solve the model in each year of the transition towards a steady state. The solution algorithm is based on the assumption that individuals have rational expectations: when making decisions they base their choices on the correct expectations of all future changes in economic conditions. Let us assume that the lenght of the transition is T years.

First iteration

1. Start with year one. Make an initial guess on the vector of the skill prices, $p_t \equiv [p_{1,t}, p_{2,t}, p_{3,t}]$, and assume that the prices from year one to the final year T are the same, that is future prices equal current ones. Also assume that individuals have perfect foresight about future skill prices: they take the vector of the skill prices p_t as given and constant in each year of their lifecycle.

2. For all ages (cohorts) take an exogenous distribution of wealth and education.

3. Given p_t and the distribution of wealth and education, simulate the behavior of samples of 10000 individuals per cohort by drawing from the distribution of the idiosyncratic shocks.

4. For each cohort, compute aggregate skill supplies and physical capital summing over individual education supply and optimal asset choices.

5. Update p_t with the market clearing prices and save the vector of equilibrium prices for this year.
6. Go to year two. The distribution of wealth and education is endogenously determined by individuals' optimal savings and education choices made in year one. Individuals in the final year of adult life die. Individuals in the first year of adult life inherit wealth and education from retirees in year one. Individuals aged in between update the values of the state space variables according to the optimal asset and education choices made in year one. Given a new initial guess on prices, repeat steps from (3) to (5) above.
7. Go on until year T repeating for each year the steps from step (1) to (6) above.
8. Collect the vectors of equilibrium prices for all years from $t = 1$ to $t = T$: this results into a $(Tx3)$ matrix that contains the sequence of the skill prices for the three education levels that represents the first iteration equilibrium.

From the second iteration onwards

9 Start with year one. Compute a new initial guess for the skill prices as a function of the guess and the value of the equilibrium prices for this year from the previous iteration.

10 Repeat all the steps from (1) to (8) above and collect the $(Tx3)$ matrix of the sequence of prices for this iteration.

11 Repeat steps (9) and (10) for successive iterations until the initial guess on the vector of prices for each year from $t = 1$ to $t = T$ is close enough to the sequence of equilibrium prices for this iteration. This is the rational expectations equilibrium where skill markets clear and individuals' expectations about future prices are realized as the equilibrium price sequence.

T is set to 50. The model always converges in less than 50 periods, so increasing the lenght of the transition does not affect the results.